# A Spatial-Temporal Statistical Approach to Problems In Command and Control

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**Abstract**

The integration, visualization, and overall management of battle-space information for the purpose of Command and Control (C2) is a challenging problem. For example, how can one assimilate incoming data rapidly in a highly dynamic environment, to allow the battle commander to make timely and informed decisions? In this paper, we present a spatial-temporal statistical approach to estimating the battlefield, based on noisy data from multiple sources. Specifically, we examine the danger-potential field generated by an enemy's weapons in the spatial domain and extend it to incorporate the temporal dimension. We propose that maps of fields of this sort are very effective decision tools for the battle commander; methods for rapid updating of the maps is an area of current research. This includes visualization of the predictions and the uncertainty associated with them. It is the quantification of uncertainty in C2 predictions that distinguishes our statistical approach from deterministic approaches. In this paper we describe an object-oriented combat-simulation program from which we generate noisy battlefield data; in the absence of real data, we apply our methodology to these simulated data.

## 1. A Brief Introduction to C2

Command and Control (C2) and related variants (C3, C4I, etc.) are umbrella terms used to describe a body of research and applications dedicated to preparing all branches of the armed forces for the "War After Next" [2]. Applications incorporated into the greater C2 framework include, but are not limited to, command applications, operations applications, intelligence applications, fire-support applications, logistics applications, and communications applications. C2 needs are shared by all branches of the armed services. Consequently, applications should be sufficiently flexible to work across services and across allied forces, as needed. In order to be applicable to future wars, applications need to be able to convert a flood of data from a wide variety of sources into information and knowledge in a timely manner. These concepts are now taught in a focused way by the U.S. Military, during officer training [3].

In preparing for the Command Post of the Future, tools should be developed with the following three goals in mind: to rapidly visualize the battlespace, to rapidly analyze the battlespace, and to rapidly understand the battlespace. When visualizing the battlespace, the prototypical commander wants a variety of information, and urgent information needs to be highlighted so that time-critical decisions can be made. In addition, the location and status of friendly and enemy forces need to be available to the commander. In order to facilitate the rapid visualization and understanding of the battlespace, to provide the ability to receive and send information while mobile, and to begin converting that information into knowledge, is critical to the battle commander. Facility for intelligent alerting and reporting, as well as customizable tactical display elements, need to be integrated into any C2 application.

Systems developed for C2 applications need to keep several general capabilities in mind. The military currently suffers not from a lack of data but from a flood of data. Data arrive from multiple sources and possibly multiple nationalities. Data arrive in visual, verbal, and other sensor formats, and also from historical data bases. At the same time, there is a distinct deficiency of information and knowledge. C2 applications should aim at developing decision aids that can turn massive amounts of data into highly useful information and that reduce the number of viable options available to the commander at key decision-making junctures. Further, even though there are large amounts of data available, developed systems must be able to deal with missing and corrupted data. In addition to traditional causes of missing or corrupted data, military applications face the potential problem of hostile corruption. In a war, it should be anticipated that data could be actively intercepted, altered, or destroyed by unfriendly forces.

Given the wide variety of battle scenarios possible in future wars, systems developed for C2 need to be scaleable between large-scale operations and unit tactics. At the same time, these systems need to provide all war fighters with a common picture of any particular battlespace. Finally, and perhaps most crucially, information processing needs to be completed quickly. To the modern war fighter, time is of the essence and it is projected that speed of processing will become even more vital in the future.

Statisticians have a unique perspective on the challenges presented by C2 and thus have an opportunity to contribute to research and applications. Statistical techniques can help make the transition from the flood of data to meaningful information and knowledge for decision making. For example, consider the area of situational awareness. The statistician might consider probabilistic frameworks, scaleable algorithms, and sequential decision-making. In particular, one might examine methods of estimating and predicting the threat or danger to a region, posed by enemy constituents. Additional areas of interest are change and anomaly detection, measures of information and understanding, and decision theory. Statisticians should examine source data, developing methods to address confidence, accuracy, and completeness of such data. They might also consider the validity of results after information processing and the quality of database information. Other research areas should focus on the representation of uncertainty or confidence in statistical results and might involve algorithms that allow statistical inference to be decentralized. Note that, in addition to providing processing stability in a hostile environment, such decentralization should lead to increased inference speed through parallel, distributed processing.

In conclusion, Command and Control (C2) is a broad field, providing a wide variety of options for statistical research and applications. In this paper, we give particular attention to one aspect, namely optimal mapping of the regional danger posed by enemy constituents in the battlespace. In Section 2, we develop an abstraction of C2 for statistical modeling and analysis and define the spatial-temporal danger-potential field. In Section 3, we discuss the object-oriented combat-simulation program we have developed in some detail. In Section 4, we examine the data for C2 decisions and the possible degradations such data might suffer. We consider analysis of the data with regard to estimating danger-potential fields in Section 5. Finally, in Section 6, we summarize our efforts and discuss future directions.

## 2. Abstraction of C2 for Statistical Modeling and Analysis

Before considering C2 from a statistical point-of-view, it is important to examine the nature of the battlespace, D, and the data that might emerge from it, as well as the state process of interest. By the state process, or Y-process, we mean the underlying spatial-temporal process describing the battle, assuming perfect, noiseless, complete knowledge. It was decided that, regardless of the choice of Y, a flexible simulation tool would be developed to produce imperfect, noisy, incomplete data Z. To define the tool properly, a set of definitions and rules were established. In order to keep the structure as flexible as possible, we settled on the following hierarchical design for any battlespace object.

At the largest size, the battlespace was defined to be a meta-object. For a particular simulation experiment, the battlespace contains all the other objects in the hierarchy. The next smallest class of objects is made up of *constituents*. In general, a constituent is an object that cannot be further combined (in the scope of the experiment) with another object of that class in a meaningful way. Specific examples of constituents include the environment in which the battle occurs, tanks and other offensive objects, and radar towers and other passive sensing objects. It should be noted that for battles of a larger scale, a constituent might actually be a group of 'smaller' objects. For example, a group of three tanks on patrol together might be considered a constituent in a battle scenario of broad enough extent. In the scale considered in this paper, single tanks are considered

Figure 1: Hierarchical Design of Battlespace Object

meta-object
(battlespace)

↙ ↘

| physical constituents (environment) | military constituents (offensive-type, observer-type) |
|---|---|
| ↓ | ↓ |
| physical elements (natural terrain, urban terrain, landmark, weather) | military elements (sensor, weapon, movement) |
| ↙ ↘ | ↙ ↘ |
| functions (contour) — parameters (altitude) | functions (goZigZag, coneError) — parameters (velocity, variance) |

to be constituents. Constituents are made up of *elements*. Elements are a smaller class of objects that have a specific sort of activity assigned to them. A tank constituent, for example, might have one or more human elements, mobility elements, sensor elements, and weapon elements. Elements of the environment constituent might include terrain and weather. The smallest classes of objects in our hierarchy are *functions* and *parameters*. Functions and parameters work together to define specifically how elements perform their activities. Functions may have deterministic or random aspects associated with them, and parameters provide the necessary numeric arguments for functions. Again, for a movement element of a tank constituent, functions might include a deterministic 'head straight for your target' function ('goStraight' object) or a 'random walk toward your target' function ('goZigZag' object). Either of these functions might accept parameters such as maximum speed, mean angle, and the standard deviations on speed and angle. Other functions might include those describing targeting error ('coneError') or other scenario-specific activities. Functions need not necessarily be limited to acting on parameters. For instance, in more complicated models, elements of a given constituent might interact according to some element-level function. It is conceivable that even higher-level interactions or functions might also occur, but our analysis will focus only on the parameter-level functions for the time being. In the design of this hierarchy, additional object classes could be inserted for more complex simulations. For example, a *component* class could be inserted between *constituents* and *elements*.

It should be noted that one or more commanders may be associated with each level of this hierarchy. An overall battle commander would be associated with the battlespace at the meta-object level; at the military-constituent and military-element levels, a commander would receive orders from the overall commander but would also direct actions associated with that constituent or element. Even at the lowest level of the hierarchy, one can imagine a function commander who is responsible for performing the tasks described by the function and who follows orders from higher levels of the hierarchy.

At this point, a brief notational comment should be made. For the purposes of formulating the battlespace model and developing the analysis, constituents are notated as vectors. Offensive-type constituents, such as tanks, are notated as $\mathbf{w}_k$, for $k = 1, 2, \ldots$. Observer-type constituents, such as radar towers, are notated as $\mathbf{v}_i$, for $i = 1, 2, \ldots$. In initial work, this vector is interpreted strictly as the Cartesian coordinates of the constituent in question, but it can be thought of more generally as the state of the constituent in question. Elements of a particular constituent are assumed to be located at the same location as their 'parent' constituent, although this assumption could be generalized if the constituents are distributed. When time is incorporated into the analysis, these vectors become functions of $t$. Specifically, in spatial-temporal-analysis settings, offensive-type constituents are notated $\mathbf{w}_k(t)$ and observer-type constituents are notated $\mathbf{v}_i(t)$. This notation refers to the location (or state) of the constituent in question at time $t$.

Consequently, all our analyses are done on continuous fields rather than on grids or lattices, although the grid is used for some numerical and visualization algorithms. For convenience, we have started with a region that is a rectangle of fixed size. In this paper, we focus on simple, flat terrain elements, but later work will include the effects of more complex terrain elements.

To carry out a battlespace simulation, we need to define the number, position, and type of constituents that would exist within a battle. This could be done in advance or assigned randomly in an obvious way (e.g., Poisson distribution [4] for numbers of objects, constrained and scaled uniform distribution or Beta distribution [4] for location coordinates). For initial experiments, two general classes of constituents were settled on: weapons and observers. We shall discuss observers in greater detail later (see Section 4, where data collection and degradation is considered), but we note here that observers are defined by functions and parameters associated with their viewing area and their error types.

Weapons, in general, have more complicated functions and parameters than do observers. For the simplest scenarios, only functions and related parameters associated with movement and offensive elements need to be considered. For more complicated scenarios, defensive functions and parameters need to be defined as well. Movement parameters include such things as autoregressive parameters [1] and error types, assigned waypoints, and commands. Offensive functions and parameters include targeting functions (such as 'coneError') and the explosion parameters used below. Defensive parameters include the amount of damage that an element can suffer and still be operational and the amount of resistance an element carries against damage. We recognize that many additional parameters might be of interest to the experimenter, and so provision for their inclusion later was incorporated into the design of the simulation.

All functions were stipulated with an eye to keeping them and the simulation tool as flexible as possible. Some parameters, for example explosion parameters, were assumed to be specified and deterministic, while other parameters, for example instantaneous velocity, were given probability distributions. For a single time unit, the velocity parameter might be distributed as a scaled Beta random variable. Targeting is assigned a radius-angle error distribution [6]. This idea is discussed further in Section 4 but, in general, it was decided to apply error to the radius and angle from the weapon to the target, rather than to the Cartesian coordinates of the weapon-to-target displacement. For initial simulation experiments, waypoint parameters were chosen to be purely deterministic, with the idea that later they may be a random function based on commander orders and affected by environmental parameters. Waypoints are pre-specified locations in space that a given mobile object is ordered to reach at pre-specified times. One way to model randomness in waypoints is to consider the location-based error arising from landmark-based orders. That is, a constituent might be instructed to move to a specific landmark element within the natural- or urban-environment constituent, rather than to a specific set of coordinates. If the location of the landmark element is known with error, a level of uncertainty emerges.

Recognizing that maps are an intuitive way to present knowledge, we now focus on mapping some sort of summary of the Y-process based on imperfect, noisy, incomplete data, $Z$. As an example, consider the damage potential posed by an enemy unit or set of enemy units. This should be particularly interesting to a commander, and it exhibits a lot of space-time variability. From the damage potential, we wish to estimate a danger-potential field generated by a set of enemy units and to examine the uncertainties associated with it. We consider our study illustrative of the general problem of producing statistically optimal maps that evolve with the changing battlespace.

We see the damage potential of an enemy weapon as analogous to gravitational potential; that is, the damage that a weapon could do to a target, can be thought of as being equal to the damage potential of a weapon element times an armor parameter that depends on the target's ability to protect itself. Thus, the damage potential of a given weapon element is equal to the damage it could do to any target constituent with a unit armor parameter.

Clearly, the assumption that a missle applies damage at a precise confined location is unrealistic. Consequently, all damage was considered to be of an explosive type, that is, affecting a continuous region and being a non-increasing function of distance from the impact point. The following formula describes one possible form of the damage potential at a distance r from the impact point:

$$\delta(r) = \begin{cases} \alpha(1-(r/R)^{p_1})^{p_2}, & \text{for} \quad 0 < r < R \\ 0, & \text{else}, \end{cases} \tag{1}$$

where $\alpha$, $R$, $p_1$, and $p_2$ are all explosion parameters defined for the weapon element in question. Under this definition, a single location in the battlespace can be affected by damage resulting from nearby impacts in the space, and the damage potential will vary with the distance from the impact.

Before continuing, consider the following notational conventions. Specifically, let $\mathbf{w}_{kl}$ denote the $l$th location impacted by weapon $\mathbf{w}_k$, allowing a single weapon to have possible multiple 'hits' in a short, specific time interval. For the purpose of this paper, attention will be focused on single-impact weapons with the impact location denoted $\mathbf{w}_{k1}$. Further, denote $f(\mathbf{w}_{k1}|\mathbf{s}, \mathbf{w}_k)$ as the probability density function of an impact at location $\mathbf{w}_{k1}$ given the weapon is located at $\mathbf{w}_k$ and is aiming at location $\mathbf{s}$. Similarly, let $\mathbf{v}_{ij}$ denote the $j$th location observed by observer $i$ in a specific time interval. Notice that enemy weapon constituents may not always be distinguishable by an observer and, thus, we do not know in general to which weapon each $\mathbf{v}_{ij}$ refers. However, for the purpose of this paper, we shall assume that the observe can identify the weapon without ambiguity and we thus let $\mathbf{v}_{ik}$ denote the location observed by observer $i$ for weapon $k$.

We define the *danger potential*, generated by a single weapon element at location $\mathbf{w}_k$, as the expected damage at any location $\mathbf{s}$:

$$g(\mathbf{s}; \mathbf{w}_k) = \int_{\mathbf{w}_{k1}} \delta(r_{\mathbf{s},\mathbf{w}_{k1}}) f(\mathbf{w}_{k1}|\mathbf{s}, \mathbf{w}_k) d\mathbf{w}_{k1}; \qquad \mathbf{s} \in D. \tag{2}$$

The distribution, $f(\mathbf{w}_{k1}|\mathbf{s}, \mathbf{w}_k)$, is based on the same radius-angle probability distributions given in Section 4. It should be noted that, given the parameters of the $k$-th weapon, $g(\mathbf{s}; \mathbf{w}_k)$ can be computed ahead of time, thus reducing the amount of time it takes to generate a danger-potential map over D.

We further assume that danger is summable. That is, we define the danger potential to an object at a specific location, from a set of enemy weapon elements, as the sum of the individual danger potentials of each weapon element. Such a definition makes sense if the individual damage potentials are summable, as we now illustrate. The danger potential at $\mathbf{s}$ is,

$$\begin{aligned} g(\mathbf{s}; \{\mathbf{w}_k\}) &= \int_{\mathbf{w}_{k1}} \sum_k \delta(r_{s,\mathbf{w}_{k1}}) f(\mathbf{w}_{k1}|\mathbf{s}, \mathbf{w}_k) d\mathbf{w}_{k1} && (3) \\ &= \sum_k \int_{\mathbf{w}_{k1}} \delta(r_{s,\mathbf{w}_{k1}}) f(\mathbf{w}_{k1}|\mathbf{s}, \mathbf{w}_k) d\mathbf{w}_{k1} && (4) \\ &= \sum_k g(\mathbf{s}; \mathbf{w}_k), && (5) \end{aligned}$$

where $\sum_k \delta(r_{s,\mathbf{w}_{k1}})$ is the damage potential of the multiple weapons $\{\mathbf{w}_{k1}\}$.

Though danger potential $g(\mathbf{s};\mathbf{w}_k)$ was defined above in a purely spatial setting, there is a natural extension to the spatial-temporal setting. Consider an offensive constituent at location $\mathbf{w}_k(\tau)$ and time $\tau$. Then the expected damage potential at location $\mathbf{s}$ and time $t > \tau$, not only depends on the probability of applying damage, but also on the probability of the weapon's location at unobserved times. That is, in addition to taking an expectation over the targeting distribution, as in (2), the expectation of the damage at a given spatial-temporal location is also based on knowledge of the weapon's location at some previous time ($\tau$). The resulting danger-potential field at $\mathbf{s}$ and $t$ is:

$$\begin{aligned}
g(\mathbf{s},t;\mathbf{w}_k(\tau),\tau) &= \int_{\mathbf{w}_{k1}(t)} \delta(r_{\mathbf{s},\mathbf{w}_{k1}(t),t}) f_1(\mathbf{w}_{k1}(t)|\mathbf{s},t,\mathbf{w}_k(\tau),\tau)d\mathbf{w}_{k1}(t) && (6)\\
&= \int_{\mathbf{w}_{k1}(t)} \delta(r_{\mathbf{s},\mathbf{w}_{k1}(t),t}) && (7)\\
&\quad \times \int_{\mathbf{w}_k(t)} f_2(\mathbf{w}_{k1}(t),\mathbf{w}_k(t)|\mathbf{s},t,\mathbf{w}_k(\tau),\tau)d\mathbf{w}_k(t)d\mathbf{w}_{k1}(t)\\
&= \int_{\mathbf{w}_{k1}(t)} \int_{\mathbf{w}_k(t)} \delta(r_{\mathbf{s},\mathbf{w}_{k1}(t),t}) f_3(\mathbf{w}_{k1}(t)|\mathbf{s},t,\mathbf{w}_k(t),t)\\
&\quad \times h_1(\mathbf{w}_k(t)|t,\mathbf{w}_k(\tau),\tau)d\mathbf{w}_k(t)d\mathbf{w}_{k1}(t) && (8)\\
&= \int_{\mathbf{w}_k(t)} g(\mathbf{s},t;\mathbf{w}_k(t),t)h_1(\mathbf{w}_k(t)|t,\mathbf{w}_k(\tau),\tau)d\mathbf{w}_k(t)\,. && (9)
\end{aligned}$$

The probability density function, $h_1(\mathbf{w}_k(t)|t,\mathbf{w}_k(\tau),\tau)$, represents the conditional probability that the weapon is at $\mathbf{w}_k(t)$ at time $t$ given that it was at spatial-temporal location $(\mathbf{w}_k(\tau),\tau)$, and it introduces a dynamic aspect to the analysis. These equalities hold assuming that danger is instantaneous, that the old location has no effect on targeting if the current location is known, and that the movement of the weapon does not depend on the generic spatial index $\mathbf{s}$. The probability depends not only on the movement function and the parameters associated with the weapon element, but also on the evolution of the spatial-temporal battlespace. For instance, if the weapon element is damaged, its mobility may be affected. In the simple example discussed in this paper, the weapon cannot be damaged, so that part of the dynamic aspect may be ignored. Expanding this conceptualization, one might re-write (8) and (9) as:

$$\begin{aligned}
g(\mathbf{s},t;\mathbf{W}) &= \sum_k \int_{\mathbf{w}_{k1}(t)} \int_{\mathbf{w}_k(t)} \delta(r_{\mathbf{s},\mathbf{w}_{k1}(t),t}) f_3(\mathbf{w}_{k1}(t)|\mathbf{s},t,\mathbf{w}_k(t),t)\\
&\quad \times h_2(\mathbf{w}_k(t)|t,\mathbf{W})d\mathbf{w}_k(t)d\mathbf{w}_{k1}(t) && (10)\\
&= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s},t;\mathbf{w}_k(t),t)h_2(\mathbf{w}_k(t)|t,\mathbf{W})d\mathbf{w}_k(t)\,, && (11)
\end{aligned}$$

where $\mathbf{W} \equiv \{(\mathbf{w}_k(\tau), \tau)\colon \tau < t\,;\; k = 1, 2, \ldots\}$ is the set of all past information on all weapons and $h_2(\mathbf{w}_k(t)|t, \mathbf{W})$ is the conditional probability density function describing the probability that a weapon is at a specific location in space-time, given knowledge of all weapons at various locations in space-time.

Figure 2: Notation Conventions

| | |
|---|---|
| $\mathbf{v}_i, \mathbf{v}_i(t)$ | The location (state) of observer $i$ (at time $t$). |
| $\mathbf{v}_{ik}, \mathbf{v}_{ik}(t)$ | The location of weapon $k$ observed by observer $i$ (at time $t$). |
| $\mathbf{w}_k, \mathbf{w}_k(t)$ | The true location (state) of weapon $k$ (at time $t$). |
| $\mathbf{w}_{kl}, \mathbf{w}_{kl}(t)$ | The $l^{th}$ location impacted by weapon $k$ (at time $t$). |
| $\mathbf{s}, \mathbf{s}(t)$ | A hypothetical location (at time $t$). |
| $r_{\mathbf{s}\mathbf{w}_k}$ | The Euclidean distance, $\|\|\mathbf{w}_k - \mathbf{s}\|\|$, between weapon $k$ at $\mathbf{w}_k$ and the target location $\mathbf{s}$. |
| $r_{\mathbf{w}_k\mathbf{w}_{kl}}$ | The Euclidean distance, $\|\|\mathbf{w}_{kl} - \mathbf{w}_k\|\|$, between weapon $k$ at $\mathbf{w}_k$ and the impact location $\mathbf{w}_{kl}$. |
| $r_{\mathbf{s}\mathbf{w}_{kl}}$ | The Euclidean distance, $\|\|\mathbf{w}_{kl} - \mathbf{s}\|\|$, between the impact location $\mathbf{w}_{kl}$ and the target location $\mathbf{s}$. |
| $\theta_{\mathbf{s}\mathbf{w}_k}$ | The angle, $\arctan[(w_{ky} - s_y)/(w_{kx} - s_x)]$, between weapon $k$ at $\mathbf{w}_k$ and the target location $\mathbf{s}$. |
| $\theta_{\mathbf{w}_k\mathbf{w}_{kl}}$ | The angle, $\arctan[(w_{kly} - w_{ky})/(w_{klx} - w_{kx})]$, between weapon $k$ at $\mathbf{w}_k$ and the impact location $\mathbf{w}_{kl}$. |
| $\theta_{\mathbf{s}\mathbf{w}_{kl}}$ | The angle, $\arctan[(w_{kly} - s_y)/(w_{klx} - s_x)]$,between the impact location $\mathbf{w}_{kl}$ and the target location $\mathbf{s}$. |
| etc. | |

In the proposed conceptualization, data are collected primarily as observations of military-constituents locations. That is, at a given time point, each observer constituent provides a list of Cartesian coordinates representing its observed locations of other constituents. Note that these observed locations are almost certainly noisy and perhaps compromised through censoring or enemy interference. The nature of the error associated with these data is discussed further in Section 4.

Given the data, our goal in this article is to estimate and map the true danger potential everywhere in the spatial-temporal region of interest. Information about the parameters of the weapon constituents and observer constituents is combined to generate estimates of the danger potential. An advantage of the spatial-temporal statistical approaches discussed in this paper is the flexibility of the questions that might be answered using the data. Simple questions about the location of a weapon or a set of weapons are not the only ones that are answerable. In addition, one might pose questions such as, "How often are enemy weapons within 10 miles of the border?", or "Does the danger posed to friendly regions appear to increase significantly over time?", or "If friendly mobile objects need to go from $A$ to $B$, what is the path of least danger?", or "Where are the enemy's mobile weapons headed?" All of these questions can be answered through some *nonlinear* functional of $\{\mathbf{w}_k(\tau)\colon \tau \leq t\,;\; k = 1, 2, \ldots\}$, and thus techniques such as Kalman filtering [8] will typically yield biased estimates. An area of research for us is to develop smoothers/filters/forecasts for which any nonlinear functional (e.g., danger-potential field) is approximately unbiased.

### 3. Object-Oriented Combat-Simulation Program

A battle simulation program has been writen in the interpreted, object-oriented language R (`http://www.stat.cmu.edu/R/CRAN/`). The R program implements a dialect of the programing language S, developed at Bell Laboratories.

We can think about the battlespace as consisting of military *constituents* (e.g., tanks, radar stations, etc.) that interact and change in time in surrounding physical constituents (the environment). The constituents are constructed in a hierarchical fashion, using *elements* with specific functionality (e.g., a gun is one element of a tank, a sensor is another).

The simulation program consists of defining different *classes* of constituents and elements, and writing functions that act on both constituents and elements (e.g., a tank has a gun element with functionality defined by its parameters, which include range, accuracy, and explosive power).

The simulation consists of an inner and an outer loop. The outer one loops through time and the inner one loops through the military constituents (we have assumed for the moment that the physical constituents are constant during the period of simulation).

The inner loop consists of the following major steps:

**Scanning the Battlespace: The scanBS Function.** Most constituents have some way of observing the battlespace in order to detect other constituents. Currently, a constituent can have any number of elements of the class sensor.

**Making a Decision: The askCommander Function.** Every constituent has a commander that makes decisions concerning the current destination (if the constituent is mobile) and applying weapons (if available). The commander of a mobile constituent has, in most cases, a set of waypoints to follow to reach its final destination. At the same time, the commander is receiving information from the sensors about the status of the surrounding battlespace at every time point. Depending on the 'character' (e.g., aggressive, defensive, etc.) of the commander, a decision is made.

**Change Location: The move Function.** A constituent moves according to its current speed and destination (both determined by the constituent commander). There are a number of ways a constituent can move at each time-step, including a method that assigns a little random error to both speed and direction at each time-step.

**Attack: The attack Function.** A constituent can have one or more elements of the class weapon. Applying a weapon consists of (1) shooting at a given target, but with error, and (2) evaluating the damage caused by the weapon, to all constituents (including the target) in the battlespace within damage range.

**Update Attributes: The update Function.** Keeps track of the history by collecting, at each time-step, parameters of interest and storing them (e.g., the value of the life and armor parameters; see Section 3.4)

Some of these major steps do not necessarily apply to all constituents.

Currently, two classes of millitary constituents have been defined. The basic constituent is a stationary object of *class* defObj with a commander. An extension of the defObj class is the mobileObj. The mobileObj *inherits* all the attributes and functions from the defObj. The only addition to the defObj is a function and parameters to make the object mobile.

Two classes of elements have currently been defined, namely, sensor and weapon. An object of class sensor has a function to observe what is in its surroundings; that is, it observes the position of natural and military constituents within the sensor's range, but with error. An element of class weapon has a function to "shoot", with error, at a given target and functions to compute the damage caused to other constituents using their life and armor values. All these error distributions are of the radius-angle type, where the radius distribution is assumed independent of the angle distribution. Furthermore, the *radius* distribution is always *truncated* so that no random radius is bigger than $r_{\max}$, and the *angle* distribution is always *wrapped* so that no random angle is outsie $(-\pi, \pi]$.

Other constituent classes can be created by extending the two basic classes; for example, the constituent class, tank, has been specified by extending the mobileObj class and adding a radar (a sensor with specific sensor function) and a gun (weapon object with attributes specified to model the workings of a gun on a tank).

### 3.1 Sensor

Any military constituent can have one or more elements of class sensor. Each sensor element checks for constituents within the range of the sensor. Each sensor is provided with a sensor function and a list of parameters that define the procedure used to generate noisy observations of the constitutents' location within the sensor's range. The sensor function can be easily adapted to diverse types of sensors.

**coneErrorSensor** Let $r_{\mathbf{v}_i\mathbf{w}_k}$ and $\theta_{\mathbf{v}_i\mathbf{w}_k}$ be the true distance and angle, respectively, between the sensor and the observed constituent, with respect to the sensor. If $r_{\mathbf{v}_i\mathbf{w}_k}$ is within the sensor's range, then suppose we observe $(r_{\mathbf{v}_i\mathbf{v}_{ik}}, \theta_{\mathbf{v}_i\mathbf{v}_{ik}})$ and express the error of that observation through:

$$r_{\mathbf{v}_i\mathbf{v}_{ik}} = r_{\mathbf{v}_i\mathbf{w}_k} e^{\varepsilon_r} \quad \text{and} \quad \theta_{\mathbf{v}_i\mathbf{v}_{ik}} = \theta_{\mathbf{v}_i\mathbf{w}_k} + \varepsilon_\theta, \tag{12}$$

where

$$\varepsilon_r \quad \sim Gau(mean = 0\,,\ variance = \sigma_r^2), \tag{13}$$

$$\varepsilon_\theta \quad \sim Gau(mean = 0\,,\ variance = \sigma_\theta^2)\,, \tag{14}$$

and when necessary truncation and wrapping occurs in (12). Then a realization of the constituent's location, $\mathbf{v}_{ik} = (v_{ikx}, v_{iky})$, is given by $v_{ikx} = v_{ix} + r_{\mathbf{v}_i\mathbf{v}_{ik}} \cos(\theta_{\mathbf{v}_i\mathbf{v}_{ik}})$ and $v_{iky} = v_{iy} + r_{\mathbf{v}_i\mathbf{v}_{ik}} \sin(\theta_{\mathbf{v}_i\mathbf{v}_{ik}})$, where $\mathbf{v}_i = (v_{ix}, v_{iy})$ is the location of the sensor.

**bivarNormSensor** If $r_{\mathbf{v}_i\mathbf{w}_k}$ is within the sensor's range and $\mathbf{w}_k = (w_{kx}, w_{ky})$ is the true location of the weapon constituent, then suppose we observe $\mathbf{v}_{ik} = (v_{ikx}, v_{iky})$ and express the error of the observation through:

$$v_{ikx} = w_{kx} + \varepsilon_x \quad and \quad v_{iky} = w_{ky} + \varepsilon_y, \tag{15}$$

where independently,

$$\varepsilon_x, \varepsilon_y \sim Gau(mean = 0, variance = \sigma^2)\,. \tag{16}$$

### 3.2 The Commander

Every class of constituent has a commander, who we shall call the constituent commander. For a constituent of the class, defObj, the simplest commander does nothing. For a mobile constituent of the class, mobileObj, the simplest commander just follows the waypoints given to reach its final destination. In general, a commander element is a function that alters the movement of the constituent and gives weapons a target to shoot at.

Other types of commanders that have been constructed include an aggresiveCommander and a defensiveCommander. The aggressiveCommander chases and then attacks every enemy seen on any sensor, but the defensiveCommander sticks to its given waypoints and shoots at any enemy within range (but does not chase them).

### 3.3 Mobility

For a constituent of the class, mobileObj, and all other classes that inherit the mobileObj class, there is a function that changes the position of the constituent at each time-step.

Different functions can be specifed as to how the constituent can accomplish this. In general, if $s_t$ is the speed of the constituent at time $t$ and $\phi_t$ its angle of trajectory, then the speed and angle at time $t+1$ are given by

$$\begin{aligned} s_{t+1} &= \rho_s s_t + (1-\rho_s) S_{t+1}, & (17)\\ \phi_{t+1} &= \rho_a \phi_t + (1-\rho_a)\Phi_{t+1}, & (18) \end{aligned}$$

where the $\rho_s \in [0,1]$ and $\rho_a \in [0,1]$ are given, and $S_{t+1}$ and $\Phi_{t+1}$ are given by a move function. A number of move functions have been constructed. The two most frequently used are:

**goStraightLNorm** There is no error involved in the angle, only in the speed:

$$\begin{aligned} \Phi_{t+1} &= \phi, & (19)\\ log(S_{t+1}) &\sim Gau(mean = \mu_s,\ variance = \sigma_s^2). & (20) \end{aligned}$$

**goZigZag** In this case, there is error in both the angle and the speed:

$$\Phi_{t+1} \sim Gau(mean = \phi, variance = \sigma_{\phi}^2), \tag{21}$$

$$log(S_{t+1}) \sim Gau(mean = \mu_s,\ variance = \sigma_s^2)\,, \tag{22}$$

and when necessary wrapping occurs in (21).

### 3.4 Weapons

A military constituent can have any number of weapon elements. Other constituents provide targets for the weapons.

Using a weapon consists of four major parts (functions):

**Shooting:** The weapon has a function, and a list of parameters, that specify how the actual impact point of the weapon is computed given the target position and the position of the weapon.

**Damage:** At the impact point (provided by the shooting function), a damage factor (potential damage) to the constituents within the maximum damage range are computed by the damage function.

**Life Reduction:** The damage factors are used by the life function to reduce the life value of the constituents within the maximum damage range.

**Armor Reduction:** Similarly, the damage factors are used by the armor function to reduce the armor value of the constituents within maximum damage range.

We now give more details on the four major steps described above. Let $\mathbf{w}_k = (w_{kx}, w_{ky})$ and $\mathbf{s} = (s_x, s_y)$ denote the position of the weapon and the target, respectively, and let $r_{\mathbf{w}_k\mathbf{s}}$ and $\theta_{\mathbf{w}_k\mathbf{s}}$ denote the distance and angle of the target with respect to the weapon. The two most frequently used shooting functions are:

**coneShooting:** The impact point is given by

$$w_{k1x} = w_{kx} + (r_{\mathbf{w}_k\mathbf{s}} e^{\varepsilon_r}) \cos(\theta_{\mathbf{w}_k\mathbf{s}} + \varepsilon_\theta), \tag{23}$$

$$w_{k1y} = w_{ky} + (r_{\mathbf{w}_k\mathbf{s}} e^{\varepsilon_r}) \sin(\theta_{\mathbf{w}_k\mathbf{s}} + \varepsilon_\theta), \tag{24}$$

where independently,

$$\varepsilon_r \sim Gau(mean = 0,\ variance = \sigma_r^2), \tag{25}$$

$$\varepsilon_\theta \sim Gau(mean = 0,\ variance = \sigma_\theta^2), \tag{26}$$

and when necessary truncation occurs with $(r_{\mathbf{w}_k\mathbf{s}} e^{\varepsilon_r})$ and wrapping occurs with $(\theta_{\mathbf{w}_k\mathbf{s}} + \varepsilon_\theta)$.

**bivarNormShooting:** The impact point is given by

$$w_{k1x} = w_{kx} + \Delta x_{\mathbf{w}_k\mathbf{s}}, \tag{27}$$
$$w_{k1y} = w_{ky} + \Delta y_{\mathbf{w}_k\mathbf{s}}, \tag{28}$$

where independently,

$$\Delta x_{\mathbf{w}_k\mathbf{s}} \sim Gau(mean = s_x - w_{kx}, SD = \alpha + \beta r_{\mathbf{w}_k\mathbf{s}}), \tag{29}$$
$$\Delta y_{\mathbf{w}_k\mathbf{s}} \sim Gau(mean = s_y - w_{ky}, SD = \alpha + \beta r_{\mathbf{w}_k\mathbf{s}}), \tag{30}$$

with $\alpha$ and $\beta$ given. Note that impact is written in terms of the location of the firing weapon plus a deviation, where the deviation is based on the target location $\mathbf{s}$. The function assumes that the further the target is from the weapon, the larger the variance of the impact location.

Recall that after firing a weapon aimed at $\mathbf{s}$, the explosion actually occurs at $\mathbf{w}_{k1}$. Let $r_{\mathbf{s}\mathbf{w}_{k1}}$ denote the distance between the location of the potential target and the explosion location caused by the weapon, let $\alpha$ denote the maximum damage that the weapon can cause, and let $R$ denote the maximum damage range from the point-of-impact. The function that is currently used to define the damage is given below.

**powerDamageScaling:** The damage-factor function, $\delta(r_{\mathbf{s}\mathbf{w}_{k1}})$, is given by,

$$\delta(r_{\mathbf{s}\mathbf{w}_{k1}}) = \begin{cases} \alpha(1 - (r_{\mathbf{s}\mathbf{w}_{k1}}/R)^{p_1})^{p_2} & \text{for } 0 < r_{\mathbf{s}\mathbf{w}_{k1}} < R \\ 0 & \text{else}\,, \end{cases} \tag{31}$$

Let $\ell_c$ denote the current value of the life parameter of the constituent in question, $\ell_n$ the new life value after weapon impact, $a_c$ the value of the current armor strength of the constituent, and $\delta$ the damage factor from the explosion at the location of the constituent (as given by the damage-factor function). The current life-reduction function is defined as follows.

**scalingLifeReduction:** The new life value is given by,

$$\ell_n = \ell_c - \delta/a_c; \tag{32}$$

if $\ell_n \leq 0$, the constituent is considered 'dead' (destroyed).

The current armor-reduction function is defined as follows.

**negativeArmorReduction**: The new armor value, $a_n$, is given by

$$a_n = a_c - \delta; \tag{33}$$

if $a_n < 0$, then $a_n$ is put equal to 0.

The program allows for user-defined functions and is designed in such a way that it is easy to add new functions to elements.

### 3.5 The Environment

In the current version, a very simple physical environment is assumed; a flat terrain with no obstacles. This can be generalized; the object-oriented simulation program is very flexible.

### 3.6 Running the Program

Running the simulation consists of constructing all military constituents, which can include both offensive-type and observer-type constituents. After the simulation, each constituent stores its history, including the exact position of the constituent at all time points. Similarly, the sensor elements (if any are installed) store everything that they see. It is also possible to construct observer-type constituents after the simulation has run, and let them observe the battlespace at some specific time points, or at all time points. An example of five mobile tanks and three sensors (two radar and one satellite) in the battlespace is given in [9].

## 4. Data for C2 Decisions

Once a simulation of the Y-process has been completed, it is time to consider the generation of the data Z. Running the simulation gives the experimenter access to the true battle information but, in real-world situations, the information is degraded in some manner. To reflect this, we degraded the simulated Y-process in several ways. Censoring might occur in both the time- and space-domains for various reasons. Terrain features might limit the observation region of an observer, and technical difficulties might prevent or delay observation of data. Location-based error was also deemed to be a fairly likely situation. Less likely, but worth considering, was false data provided by the enemy.

Spatial and temporal censoring can occur under a variety of circumstances. Some examples might include adverse sensing conditions (e.g., sunspot activities, jamming) that could prevent or delay any data transmission during the affected time period. Such censoring might be generated by using exponentially distributed start, stop, and transmission delay times. It is also likely that certain sensors may not transmit continuously. These sensors may transmit at systematic intervals (for instance, one data report every 30 seconds) or at event-invoked times. One might consider a reliability probability for such sensors. That is, for a predetermined set of time points, there is a probability that at each of these times the sensor will transmit its data. Additionally, any given sensor might fail during the course of the battle. Spatial censoring might occur as the result of terrain (e.g., a hill) or meterological (e.g., clouds) effects. Finally, there may be systematic spatial censoring resulting from spatial regions outside of the sensing range of all available sensor constituents or elements. Any or all of these types of censoring may occur in a single battle scenario.

Location-based error is worth particular mention. It seems to us to be ubiquitous in all battle scenarios. Moreover the radius-angle error distributions turn up repeatedly. While standard bivariate normal error might be applied to any location (and is indeed applied to satellite-type sensors), radar-type sensors and weapon-target applications were given radius-angle error distributions.

Let $(x, y)$ be the true Cartesian coordinates of the observer location (assume known) and $(x_0, y_0)$ the true Cartesian coordinates of the location of interest (unknown). Notate the true angle and radius from the observer location to the location of interest as $\theta_0$ and $r_0$, respectively. Now suppose that $\widehat{\theta}$ and $\widehat{r}$ are independent, random, noisy observations of $\theta_0$ and $r_0$. If $\widehat{\theta}$ and $\widehat{r}$ are unbiased for $\theta_0$ and $r_0$, the nonlinear relationship between $(\theta_0, r_0)$ and $(x_0, y_0)$ leads to a bias in $(\widehat{x}, \widehat{y})$, the estimates of $(x_0, y_0)$ based on $(\widehat{\theta}, \widehat{r})$. (To see this, note that $E(\widehat{x}) = E(x + \widehat{r} \cos \widehat{\theta}) = x + E(\widehat{r})E(\cos \widehat{\theta}) = x + r_0 E(\cos \widehat{\theta})$. But $E(\cos \widehat{\theta}) \neq \cos \theta_0$, so $E(\widehat{x}) \neq x_0 = x + r_0 \cos \theta_0$. This also holds for $y_0$, by symmetry.) However, a Taylor approximation such as the one described in Section 5 below, shows that by adjusting the mean of $\widehat{r}$, an approximately unbiased estimate for $(x_0, y_0)$ can be achieved. That is, assume that the reported angle $\widehat{\theta}$ is:

$$\widehat{\theta} = \theta_0 + \varepsilon_\theta \,, \tag{34}$$

where $\varepsilon_\theta$ is distributed with mean 0 and variance $\sigma_\theta^2$; and assume that the reported distance $\widehat{r}$ is randomly distributed with (adjusted) mean $r_0/(1 - \frac{1}{2}\sigma_\theta^2)$ and variance $\sigma_r^2$. As is shown in (**??**)-(**??**) of Section 5, these assumptions lead to an approximately unbiased estimate of $(x_0, y_0)$. The angle $\widehat{\theta}$ and radius $\widehat{r}$ are then converted back into the standard Cartesian coordinates for the purpose of mapping the results.

In our simulations, a small set of distributions was considered for the radius and the angle; we always assume the radius distribution and the angle distribution to be independent. For the radius, we considered the truncated lognormal, truncated gamma, and scaled beta distributions; these were chosen because it was felt that in each case a maximum range seemed applicable. For the angle, only two distributions were considered, namely, the wrapped normal and the Von Mises distribution [6]. The default distributions for the radius-angle error distributions were the truncated lognormal for the radius, independent of the wrapped normal for the angle.

## 5. Analysis

For the purpose of this paper, the goal of the analysis is to estimate the true danger-potential field of a battlespace over time. Note that the deterministic danger-potential field is well defined if the impact-location distribution and the location of the firing weapon is known. Since the former is likely to be known from intelligence sources, our focus is on the latter. Consequently, we wish to estimate the true danger potential in the face of unknown weapon locations. We discuss two possible approaches, whose differences we shall investigate in the future. One approach is to apply Bayesian methods [5]. In this case, one might generate the distribution of danger-potential fields implied by the distribution of weapon locations. An alternate, much faster approach is to apply some sort of 'plug-in' estimation for the locations of the offensive constituent(s).

Regardless of the approach used, an estimate of the posterior probability distribution of the locations of the weapons, given the observations, would be useful. When dealing with the radius-angle errors described earlier, the distributions of the observations can be complicated to represent. However, second-degree Taylor approximations can be used to approximate the mean and the $(2 \times 2)$ covariance matrix of the observations, as we shall now demonstrate.

Before examining this approximation, the following facts are worth noting. Second-degree Taylor approximations show that, for $\varepsilon_\theta$ small, $\cos(\varepsilon_\theta)$ is approximately equal to $1 - \frac{1}{2}\varepsilon_\theta^2$, and $\sin(\varepsilon_\theta)$ is approximately equal to $\varepsilon_\theta$. Similarly, $\cos^2(\varepsilon_\theta)$, $\sin^2(\varepsilon_\theta)$, and $\cos(\varepsilon_\theta)\sin(\varepsilon_\theta)$ are each approximately equal to $1 - \varepsilon_\theta^2$, $\varepsilon_\theta^2$, and $\varepsilon_\theta$, respectively. Assuming the same distribution on the distance and angle as discussed above, consider the observation, $\mathbf{v}_{ik} \equiv (v_{ikx}, v_{iky})$. Write $\mathbf{v}_{ik}$ as $\mathbf{v}_i + \mathbf{\Delta}_{\mathbf{v}_{ik}}$, where $\mathbf{\Delta}_{\mathbf{v}_{ik}} = (\Delta_x, \Delta_y)$, $\Delta_x = r\cos(\theta)$, $\Delta_y = r\sin(\theta)$, and $\theta = \theta_{\mathbf{v}_i\mathbf{w}_k} + \epsilon_\theta$. Combining these equalities and assuming independence of $r$ and $\theta$, we obtain

$$
\begin{aligned}
E[v_{ikx}] &= E[v_{ix} + r\cos(\theta_{\mathbf{v}_i\mathbf{w}_k} + \epsilon_\theta)] \\
&= v_{ix} + E[r]E[\cos(\theta_{\mathbf{v}_i\mathbf{w}_k} + \epsilon_\theta)] \qquad (35) \\
&= v_{ix} + \frac{r_{\mathbf{v}_i\mathbf{w}_k}}{1 - \frac{1}{2}\sigma_\theta^2}\left\{\cos(\theta_{\mathbf{v}_i\mathbf{w}_k})E[\cos(\epsilon_\theta)] - \sin(\theta_{\mathbf{v}_i\mathbf{w}_k})E[\sin(\epsilon_\theta)]\right\} ,
\end{aligned}
$$

where the last equality is a consequence of the assumptions following equation (**??**). Applying the Taylor approximations, we obtain

$$
\begin{aligned}
E[v_{ikx}] &\simeq v_{ix} + \frac{r_{\mathbf{v}_i\mathbf{w}_k}}{1 - \frac{1}{2}\sigma_\theta^2}[\cos(\theta_{\mathbf{v}_i\mathbf{w}_k})E[1 - \frac{1}{2}\epsilon_\theta^2] \qquad (36) \\
&\quad - r_{\mathbf{v}_i\mathbf{w}_k}\sin(\theta_{\mathbf{v}_i\mathbf{w}_k})E[\epsilon_\theta]] \\
&= v_{ix} + r_{\mathbf{v}_i\mathbf{w}_k}\cos(\theta_{\mathbf{v}_i\mathbf{w}_k}) \qquad (37) \\
&= w_{kx} .
\end{aligned}
$$

A similar approach can be used to compute the expected values of $v_{iky}$, $v_{ikx}^2$, $v_{iky}^2$ and $v_{ikx}v_{iky}$. With these expected values, the approximate expected value and covariance matrix of $\mathbf{v}_{ik}$ can be computed, yielding

$$
E[\mathbf{v}_{ik}] \simeq \mathbf{w}_k , \qquad (38)
$$

and

$$
\mathrm{var}[\mathbf{v}_{ik}] \simeq [\sigma_r^2 - r_{\mathbf{v}_i\mathbf{w}_k}^2(1-\gamma) - \sigma_\theta^2(\sigma_r^2 + r_{\mathbf{v}_i\mathbf{w}_k}^2\gamma)]\mathbf{p}\mathbf{p}' + [\sigma_r^2 + r_{\mathbf{v}_i\mathbf{w}_k}^2\gamma]\sigma_\theta^2\mathbf{q}\mathbf{q}', \qquad (39)
$$

where $\mathbf{p} \equiv (\cos\theta_{\mathbf{v}_i\mathbf{w}_k}, \sin\theta_{\mathbf{v}_i\mathbf{w}_k})'$, $\mathbf{q} \equiv (\sin\theta_{\mathbf{v}_i\mathbf{w}_k}, -\cos\theta_{\mathbf{v}_i\mathbf{w}_k})'$, and $\gamma = (1 - \frac{1}{2}\sigma_\theta^2)^{-2}$. Note that the covariance matrix above involves the unknown angle $\theta_{\mathbf{v}_i\mathbf{w}_k}$; it is suggested that the observed angle $\theta_{\mathbf{v}_i\mathbf{v}_{ik}}$ be used in its place.

More generally, consider the following model for data obtained by observing the offensive constituent $\mathbf{w}_k$ from an observation constituent, $\mathbf{v}_i$. The resulting observation, $\mathbf{v}_{ik}$, can be expressed as:

$$\mathbf{v}_{ik} = \mathbf{w}_k + \boldsymbol{\varepsilon}_{ik}, \tag{40}$$

where $\boldsymbol{\varepsilon}_{ik}$ is a random vector with mean zero and a $(2 \times 2)$ covariance matrix depending on the type of error distribution associated with the observer, as well as the relative position of the observation constituent with respect to the offensive constituent. In the case of the observation constituents being radar sites, we assume a radius-angle error distribution and the covariance matrix of $\boldsymbol{\varepsilon}_{ik}$ is given by (**??**).

Thus, for observations $\{\mathbf{v}_{ik}: i = 1, \ldots, M\}$ on $\mathbf{w}_k$ from multiple observers, $\mathbf{v}_1, \ldots, \mathbf{v}_M$, the following model applies:

$$\begin{pmatrix} \mathbf{v}_{1k} \\ \vdots \\ \mathbf{v}_{Mk} \end{pmatrix} = G\mathbf{w}_k + \begin{pmatrix} \boldsymbol{\varepsilon}_{1k} \\ \vdots \\ \boldsymbol{\varepsilon}_{Mk} \end{pmatrix}, \tag{41}$$

where

$$G = \begin{bmatrix} I_2 \\ I_2 \\ \vdots \\ I_2 \end{bmatrix}, \tag{42}$$

and $I_2$ is the $(2\times2)$ identity matrix. The $\boldsymbol{\varepsilon}_{ik}$ in (41) are independent $(2\times1)$ zero-mean error vectors with covariance matrix, $\Sigma_i$, as given by (**??**). Notice that $G$ is a $(2M \times 2)$ matrix, $\mathbf{w}_k$ is a $(2 \times 1)$ vector, and the other vectors represented above are $(2M \times 1)$ vectors. The generalized least-squares estimator for $\mathbf{w}_k$ is then

$$\hat{\mathbf{w}}_k = (G'\Sigma^{-1}G)^{-1}G'\Sigma^{-1} \begin{bmatrix} \mathbf{v}_{1k} \\ \vdots \\ \mathbf{v}_{Mk} \end{bmatrix}, \tag{43}$$

$$\tag{44}$$

where

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_M \end{bmatrix}. \tag{45}$$

This is a rather crude estimate, and we are currently exploring filtering/forecasting methods (e.g., variants of the Kalman filter and Bayesian sequential imputation) that incorporate past information on the weapon's locations.

Recall from (11) how weapons' past locations $\mathbf{W} = \{(\mathbf{w}_k(\tau), \tau) \colon \tau < t\}$ determine the danger-potential field $g(\mathbf{s}, t; \mathbf{W})$ in space and time. We now consider a forecast of this field at the *present* time $t$, based on noisy *past* observations, $\mathbf{V} \equiv \{(\mathbf{v}_{ik}(\tau), \tau) \colon \tau < t;\ k = 1, 2, \ldots\}$ of the weapons' movements. Define

$$\begin{aligned} \widehat{g}(\mathbf{s}, t; \mathbf{V}) &\equiv E[g(\mathbf{s}, t; \mathbf{W})|\mathbf{V}] \\ &= \int_{\mathbf{W}} g(\mathbf{s}, t; \mathbf{W}) f_{\mathbf{W}|\mathbf{V}}(\mathbf{W}|\mathbf{V}) d\mathbf{W}\,, \end{aligned} \tag{46}$$

where $f_{\mathbf{W}|\mathbf{V}}(\cdot)$ is the posterior probability density of the true spatial-temporal locations $\mathbf{W}$, given the data $\mathbf{V}$. Holding all the assumptions from the development of (2)-(**??**) to be true and replacing $g(\mathbf{s}, t; \mathbf{W})$ with (**??**), (**??**) can be written as

$$\begin{aligned} \widehat{g}(\mathbf{s}, t; \mathbf{V}) &= \int_{\mathbf{W}} \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}, t; \mathbf{w}_k(t), t) h_2(\mathbf{w}_k(t)|t, \mathbf{W}) d\mathbf{w}_k(t) \\ &\qquad \times f_{\mathbf{W}|\mathbf{V}}(\mathbf{W}|\mathbf{V}) d\mathbf{W} \qquad (47) \\ &= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}, t; \mathbf{w}_k(t), t) \\ &\qquad \times \int_{\mathbf{W}} h_2(\mathbf{w}_k(t)|t, \mathbf{W}) f_{\mathbf{W}|\mathbf{V}}(\mathbf{W}|\mathbf{V}) d\mathbf{W} d\mathbf{w}_k(t)\,. \qquad (48) \end{aligned}$$

Because $\mathbf{w}_k(t)$ at $t$ and $\mathbf{V}$ are conditionally independent given $\mathbf{W}$, we can rewrite $h_2(\mathbf{w}_k(t)|t, \mathbf{W}) f_{\mathbf{W}|\mathbf{V}}(\mathbf{W}|\mathbf{V})$ as $h_3(\mathbf{w}_k(t), \mathbf{W}|t, \mathbf{V})$. Hence,

$$\begin{aligned} \widehat{g}(\mathbf{s}, t; \mathbf{V}) &= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}, t; \mathbf{w}_k(t), t) \qquad (49) \\ &\qquad \times \int_{\mathbf{W}} h_3(\mathbf{w}_k(t), \mathbf{W}|t, \mathbf{V}) d\mathbf{W} d\mathbf{w}_k(t) \\ &= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}, t; \mathbf{w}_k(t), t) h_4(\mathbf{w}_k(t)|t, \mathbf{V}) d\mathbf{w}_k(t)\,, \qquad (50) \end{aligned}$$

where $h_4(\mathbf{w}_k(t)|t, \mathbf{V})$ is the conditional probability density function describing the probability that a weapon is at a specific location at time $t$, given the past observations of all weapons at various locations. The calculations leading to (50) require modification if $\mathbf{V}$ also contains current observations $\{(\mathbf{v}_{ik}(t), t) \colon k = 1, 2, \ldots\}$; this results in a filtered (rather than a forecasted) space-time danger potential. These calculations will be reported on elsewhere. It should also be noted that at times where observations are not taken or are incomplete, the danger potential will tend to spread out as the uncertainty about the locations of the weapons increase. For an example of a space-time danger field generated by the computer program described in Section 3, see [9].

## 6. Discussion and Future Directions

In examining the ways in which Statistics can contribute meaningfully to problems in Command and Control, our research efforts at The Ohio State University [7] are based on our belief that the proper quantification of uncertainty is crucial to providing the commander with as accurate and precise information as possible. We also believe that maps are an effective way of representing the resulting knowledge and uncertainty. We have developed a hierarchical design and notation for discussing the hypothetical battlespace, which has led to the postulation of danger potential as an information tool of interest to the battle commander.

The danger-potential field has a number of desirable properties. First, the fields are summable, so that the effect of additional weapons can be easily incorporated. In addition, danger potential extends naturally to spatial-temporal fields. Since the form of the danger potential of a specific weapon can be computed in advance and can be represented concisely as a function of the weapon location, a spatial-temporal picture of the danger-potential field can be developed quickly, assuming the locations of the weapons are known.

The weapons' locations are often unknown and we are exploring two approaches to incorporating the location uncertainty into the danger-potential field: plug-in estimation and full Bayesian inference. Covariance-matching kriging methods are being considered for plug-in estimation and sequential-imputation methods are being considered for Bayesian inference.

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