ANALYSIS OF FUZZY REGRESSION FOR MODELING SHELF-LIFE OF GUN PROPELLANTS

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ABSTRACT

Shelf-life estimation of gun propellants is a situation in which representative data describing the factors influencing its behavior are not clearly established, neither is sufficient nor representative of variations presented from lot to lot. This research applies the concept of fuzzy regression analysis to model the life of the gun propellant and determines the distribution behavior that best fitted the relationship. Fuzzy regression takes into account such aspects as a non-linear relationship between factors, and variations that might exist between lots. It has been found through a simulation study that when representative data quality is inferior fuzzy regression should not be used, and instead classical statistical regression should be performed.

INTRODUCTION

This research is concern with statistically modeling the shelf-life of gun propellants. It is desire to accurately model gun propellant's life in order to predict its stability and as a result assessing how safe the gun ammunition is when it is store during peacetime. Currently, in order to get an estimate of the gun performance, information needed by the U.S. Navy, the Naval Surface Warfare Center performs a stability test of the stored gun propellants. This stability test is called *Master Sample Surveillance Program.* Data collected from it is fitted into a straight line generating a linear regression model of the stability of the gun propellant over time. In assuming that in fact there exists a linear relationship between the age of the gun propellant and the fumes it emits other external factors are ignored such as variations due to differences in characteristics among lots of propellants produced by different manufacturers, and differences that might be encountered for accelerating the aging process of the gun propellants in a heating chamber.

The assumptions made by the program are often of concern when modeling the shelf-life of gun propellants. The methods being utilized currently to fit the model assume that there in fact exists a linear relationship between the age of the propellants and the fumes it expedites. It is also observed that by making a linear regression model, variation on the stability patterns from lot to lot is not accounted.

Along with the discussion of shelf-life estimation of gun propellants it is desire to test a recently developed form of regression called fuzzy regression analysis. Which it is based in situations when boundary conditions are not concretely specified, therefore taking into account variations in the model. In the following section the fuzzy regression analysis will be further described. The next sections explain how the fuzzy regression analysis was applied to the modeling of gun shelf-life estimation and its results. As a final topic, based on the results of the application of fuzzy regression, suggestions and recommendations on the appropriateness of the model to represent gun shelf-life will be made.

FUZZY REGRESSION ANALYSIS

BACKGROUND

Fuzzy regression is a concept that aims to model situations in which estimation is influential towards conclusions made based on the system's model and when forecasting is performed in an uncertain environment[1]. Although it has been tested, compared, and proved that statistical linear regression surpasses the quality of modeling of fuzzy regression it rarely describes the true relation between variables[2]. Therefore, when making predictions on capabilities fuzzy regression offers an option to model linear regression situations in which parameters are not clearly defined or are "too complex or too ill-defined to admit for precise analysis"[3]. Its major drawback is that its quality performance is mainly influenced by such aspects as sample size, and specificity of model which are expected to match correctly with properties such as autocorrelation, and randomness of the studied sample. In these conditions is when fuzzy regression becomes a useful tool in modeling. Fuzzy regression is capable of making model predictions when such variables as reduced sample size, and unclear parameters (for example, boundaries) are considered. Overall, "fuzzy regression represents a phenomenon that is imprecise and vague in nature. In the case where many phenomena are not sharply defined, fuzzy techniques will represent these effects of causal variables in a more realistic way"[1].

A fuzzy set was initially described by Zadeh[4] which stated that samples are categorized as fuzzy when there is not a clear transition point from one sample to other. Therefore, a fuzzy linear function is one in which parameters are given by fuzzy sets. Fuzzy linear functions are intended to be modeled on the premises of Zadeh's extension principle thus when developing and/or formulating a fuzzy linear regression analysis it is intended to account for uncertainties that would not only be due to randomness, but also to fuzziness.

The main objective of fuzzy linear regression is to be a tool in modeling where a relationship among variables is not clearly defined through Zadeh's extension principle[5]. It is important to establish that fuzzy regression analysis is not a statistical method. This is because deviations in this type of modeling are due to the indefiniteness of the variables (or parameters), and not due to measurement errors. Therefore, fuzzy regression brings the opportunity to develop models where strict assumptions such as normality, natural randomness, that are associated with large sample groups can be ignored.

In taking the risk of ignoring marked characteristics of a sample, which attributes would seem not to make any significant relevance to the outcomes of the model, could attempt towards the validity and performance of the model if it is chosen to try to fit the variables in a linear pattern.

Therefore, the setting where fuzzy regression analysis is justified is when decisions involving human criteria are involved, and when a process is not clearly or patterned designed; in a situation where its output is to be predicted, with a small sample size, and with ambiguous data is available. Kim et. al[6] establishes that fuzzy regression improves over statistical regression as the size of the data set diminishes.

FUZZY REGRESSION: OBJECTIVES, DEFINITION, AND FORMULATION

Deviations when modeling fuzzy sets are accounted due to the indefiniteness of the system structure and not to errors recorded in observations influenced by human interaction. These deviations are fuzzy parameters, which set the function to be modeled to be of a vague phenomenon. The vagueness can be modeled by

$$J = c_1 + \ldots + c_n \tag{1}$$

Fuzzy regression main objective is to minimize the vagueness brought about by the fuzzy parameters. According to studies developed by Hideo Tanaka in 1982[7], fuzzy parameters are defined as:

$$\mu_A(a) = \min j\ [\mu_{Aj}\ (a_j)\ ]$$

$$\mu_A(a) = \{ 1 - | \alpha_j - a_j | / c_j \quad , \alpha_j - c_j \leq a_j < \alpha_j + c_j$$
$$0 \quad , \text{otherwise}$$
$$\text{where } c_j > 0 \tag{2}$$

Then, a fuzzy linear regression can be obtained of the form of:

$$\mathbf{Y} = \mathbf{A}_1 x_1 + \ldots + \mathbf{A}_n = \mathbf{A}x \tag{3}$$

where $\mathbf{A}_i$ have been defined to be the variable describing the fuzzy set.

If we are able to find the value of the fuzzy parameters $\mathbf{A}^*_I=(\alpha_I , c_i)$, then the basic goal of fuzzy regression, that is to reduce the vagueness introduced into the model by the uncertainty of the parameters, would be simplified to get the ith observed value of the dependent variable as close to its fuzzy estimate, defined as $y^*_I=A^*x_i$, to be at least H. H is a value between 0 and 1 that establishes the degree of fitness to which the model is desired to be represented. To minimize the fuzziness of the dependent variable the problem could be introduced as a simple linear programming problem in the form of:

$$\min \alpha, c \; J = c_1 + \ldots + c_n$$
$$\text{subject to} \quad c \geq 0,$$
$$\alpha^t x_i + (1 - H) \sum c_j \, | x_{ij} | \geq y_i + (1 - H)e_i,$$
$$\alpha^t x_i + (1 - H) \sum c_j \, | x_{ij} | \geq -y_i + (1 - H)e_i, \quad i = 1, \ldots , N \tag{4}$$

where H is the desire degree to which we would like to linearly fit the model. Values of x_i and y_i represent the inputs and outputs per observation recorded respectively. α_i is the center value and c_i its corresponding width which introduces the fuzziness and/or vagueness to the problem. It needs to be stated that the values of α are not always positive as a result $\alpha' \geq 0$ is defined to be as.

$$\alpha = \alpha' + d \tag{5}$$

It has also been stated that since the number of constraints 2N is larger than the number of variables in Eq. (4), it would be more convenient to solve it with the dual.

FUZZY REGRESSION: PARAMETERS AND APPROPRIATENESS OF THE REGRESSION

Parameters in fuzzy regression are fuzzy numbers that can be considered as a possibility distribution[9]; the estimated dependent variable is also a fuzzy number, yielding a fuzzy interval of the dependent variable[10].

It has to be noted that since fuzzy regression is a non statistical method there are non specific means to check the appropriateness of the model thus the decision maker will specify a confidence level that will be the equivalent value of H. From this, it can be concluded that the focus of the fuzzy intervals is entirely on the given observation and not on the future sample or predictions[10]. Fuzzy regression is not appropriate for predictions.

The possibility distribution shape of fuzzy model parameters usually utilizes a linear, symmetric, triangular membership function. Fuzzy regression provides for more flexibility in the shape of its distribution than statistical regression.

From several studies performed in sampling and establishing comparison between the general statistical regression processes used in the past, it has been shown that the spread of the fuzzy estimate increases with more data whereas in basic statistical regressions as more data was available the spread of the point estimator decreased. To explain the behavior of the fuzzy estimate we rely in the concept of possibility. Since the fuzzy regression algorithm requires that the fitted fuzzy model explain all possibilities in a given observation set, if the observations are increased more possibilities are to be explained by the fuzzy model. Possibility is explained but sacrificing precision.

As a summary, there are certain situations in which fuzzy regression is appropriate to be used: when observations and/or data come from fuzzy numbers, when data is obtained from measurements and/or estimated, when there is not sufficient data available to perform a study; Overall, mainly in situations when human knowledge is the source of information used for modeling.

Fuzzy linear regression usefulness could be observed in the estimation of regression parameters when there is a lack of data collected and/or aptness of the regression model is poor.

PROBLEM DEFINITION AND FUZZY REGRESSION APPLICATION

The main concern in this research is to determine an appropriate distribution to model a data set (which observations are considered fuzzy due to their uncertainty and lack of observable patterned behavior). The data to be modeled is the shelf-life of gun propellants. Since the majority of gun propellants are produced before they are completely used, its stability deteriorates during storage. Gun performance is assessed currently by the U. S. Navy through performing stability tests.

The test consists of taking several samples, five pounds each, from individual propellant lots; taking a portion of propellant of about 45 gm, from each five pound sample, which would then be heated in a chamber at 65.5°C. This heating process will accelerate the aging of the propellant causing it to generate red nitrogen oxide fumes. The time it takes to generate fumes is used as a measure of stability of the propellant once it enters the heating chamber. The general observed trend in past experiments has shown that the stability of the propellants tends to decrease in proportion with the fume time. The usefulness and safe life of the gun propellants as a result of these performed stability test is estimated by determining the fume time duration. When fume time decreases below thirty days at 65.5°C temperature the propellant is considered to be unsafe and the lot corresponding to the tested sample is destroyed.

But the method just described to determine the shelf-life of gun propellants has a few weak points. In order to obtain accurate results from this method observation of the samples to age at a 65.5°C temperature takes approximately twenty to eighty years, which is considered too long in order to correctly establish safety guidelines, do inventory management, acquisition decisions, etc. A major flaw from the testing is the assumption of linearity it makes when it is statistically represented, producing a model that it is not adequate in estimating the true shelf-life of the gun propellants. Currently, shelf-life is being modeled with fixed effects of the linear regression model, where fume time is fitted against propellant age. Data was collected from a group of similar gun propellant lots. The basic assumption taken with this regression is that there exists a linear relationship between fume time and age. The shelf-life was defined to be as the age when a "95% one-sided lower prediction limit for the fume time curve intersects the acceptable lower specification level"[11].

Three major concerns from the method just described arise, 1) How certain are we as decision makers that there in fact exists a linear relationship between stability and age of the propellant; 2) In estimating the shelf-life of gun propellants how is the lot to lot variation going to be treated; 3) Are we sure that using an accelerated condition, of submitting the gun propellant to a temperature of 65.5°C, is a reliable procedure to find unsafe propellants.

With the concerns just addressed we will try a new method of modeling situations in which variations could be taken into account, fuzzy regression analysis. As it can be seen from Table 1 the possibility distribution theory better describes a system where uncertainties are to be taken into account in modeling.

With the concerns just addressed we will try a new method of modeling situations in which variations could be

To formulate a possibility regression analysis the linear transformation in Eq. (6) is used:

$$\mathbf{Y} = \mathrm{T}\mathbf{x} \tag{6}$$

where T is defined to be a p x n matrix and rank[T]=p. For the purpose of modeling the fuzzy regression analysis **x** is going to be assumed to be governed by a possibility distribution $(\mathbf{a}, D_A)_e$.

$$\mathbf{Y} = \mathrm{T}\mathbf{A} \tag{7}$$

Defining the possibility distribution[12]

$$\prod_Y(y) = \max_{\{x|y=Tx\}} \prod_A(\mathbf{x}) \tag{8}$$

Therefore, the optimization problem would be written as[12]:

$$\min x\ (\mathbf{x} - \mathbf{a})^t D_A (\mathbf{x} - \mathbf{a})$$

$$\text{subject to} \quad \mathbf{y} = \mathrm{T}\mathbf{x} \tag{9}$$

where optimal solution are:

$$\lambda^* = 2(\mathrm{T} D_A \mathrm{T}^t)^{-1}(\mathbf{y} - \mathrm{T}\mathbf{a}),$$

$$\mathbf{x}^* = \mathbf{a} + D_A \mathrm{T}^t (\mathrm{T} D_A \mathrm{T}^t)^{-1} (\mathbf{y} - \mathrm{T}\mathbf{a}) \tag{10}$$

Defining the exponential possibility distribution as:

$$\prod_Y(y) = \exp\{-(\mathbf{y} - \mathrm{T}\mathbf{a})^t (\mathrm{T} D_A \mathrm{T}^t)^{-1} (\mathbf{y} - \mathrm{T}\mathbf{a})\},$$

$$\mathrm{Y} = (\mathrm{T}\mathbf{a}, \mathrm{T} D_A \mathrm{T}^t)_e \tag{11}$$

To specially define the possibility distribution $\prod_Y(y)$ of the output could be done by replacing T with $\mathbf{X}^t$:

$$\prod_Y(y) = \exp\{-(y - \mathbf{X}^t\mathbf{a}_c)^2 (\mathbf{X}^t D_A \mathbf{x})^{-1},$$

$$\mathrm{Y} = (\mathbf{X}^t\mathbf{a}_c, \mathbf{X}^t D_A \mathbf{x})_e \tag{12}$$

The optimization problem to find the possibility distribution can be written as:

$$\min_{D_A, a_c} J = \sum_{j=1} \mathbf{x}_j^t D \mathbf{x}_j$$

$$\text{subject to} \quad 1)\ \mathbf{x}_j^t D_A \mathbf{x}_j \geq (y - \mathbf{a}_c^t \mathbf{x}_j)^2 / (-\log h_j)$$

$$2)\ \mathbf{D}_A > 0 \tag{13}$$

The constraints presented in the optimization problem by Eq. (13) define the problem to be non-linear, thus in order to be able to use the basic procedure of problem solving of linear programming the following measures should be taken: 1) Find the center vector $\mathbf{a}_c$ through conventional regression; and 2) Substitute the obtained center vector $\mathbf{a}_c^*$ in the first constraint of the optimization problem described by Eq. (13):

$$\min_{D_A, a_c} J = \sum_{j=1} \mathbf{x}_j^t D_A \mathbf{x}_j$$

$$\text{subject to} \quad 1)\ \mathbf{x}_j^t D_A \mathbf{x}_j \geq (y - \mathbf{a}_c^{*t} \mathbf{x}_j)^2 / (-\log h_j)$$

$$2)\ \mathbf{x}_i^t D_A \mathbf{x}_i = 0 \qquad i \neq j, \qquad i,j \in E \tag{14}$$

To describe the shelf-life of gun propellants constraint "1" was modified and in the denominator $\ln h_j$ was used instead of $\log h_j$.

In order to apply the fuzzy regression analysis a data set consisting of thirty-six observations was studied. The observations recorded were of the age of the gun propellants and its associated developed fume. Observations recorded were based in a 365 days cycle, therefore for our investigative procedure observations were scaled by 365 (that is, age = age/365; fume = fume/365).

Since it is intended to make use of fuzzy regression analysis as it was explained before, this system is going to be defined as a possibility linear system of the form:

$$Y = A_1 x_1 + \ldots + A_n x_n = \mathbf{A}\mathbf{x} \tag{15}$$

Defining **A** as a fuzzy coefficient vector defined by the exponential possibility distribution "with center vector $\mathbf{a}_c$ and a symmetrical positive definite matrix D_A"[11] which takes the following form:

$$\mathbf{A} = (\mathbf{a}_c, D_A)_e \tag{16}$$

The independent variable to be modeled in this experiment was defined to be the age of the gun propellant while the fumes generated represented the dependent variable. Using conventional regression analysis the center vector $\mathbf{a}_c^*$ was obtained to be:

$$\mathbf{a}_c^* = (22.6496, -5.0068) \tag{17}$$

Although it did not seem to reflect a great impact in the ultimate result of our minimization problem, in order to establish a fair comparison with others already applied modeling systems for the gun propellant's shelf-life the center vector values utilized before were also used here:

$$\mathbf{a}_c^* = (3.97535, -0.15136) \tag{18}$$

The linear programming problem defined by Eq. (14) was solved by making use of the LP software "LINDO" by selecting ten random orthogonal condition combinations, since there are ${}_{36}C_2 = (36*35)/2 = 630$ combinations that would result in too many combinations for the experiment. Table 3 shows the (input – output) data given where $\mathbf{x}_j$ is represented as an input vector of the form $(1, x_j)^t$.

In this problem Z is defined to be as the covariance, therefore its value could be assumed as either positive and/or negative. "LINDO" only defines its variables to be nonnegative; Therefore, to account for this the Z variable value will be stated as $Z = Z_0 - Z_1$. An h_j value was set at a value of "0.5" to account for a fair confidence interval.

Then the objective function to be minimized is defined by the following function:

$$\text{Min} \quad J = 36X + 162.03998Y + 202.87Z$$

subject to

$$\begin{aligned}
&X + 6.12Y + 9.37Z \geq 7.63\\
&X + 5.49Y + 7.54Z \geq 0.91\\
&X + 4.06Y + 4.12Z \geq 3.93\\
&X + 3.77Y + 3.55Z \geq 0.21\\
&X + 3.89Y + 3.78Z \geq 4.99\\
&X + 3.17Y + 2.51Z \geq 8.01\\
&X + 5.90\,Y + 8.71Z \geq 13.16\\
&X + 5.72Y + 8.17Z \geq 0.39\\
&X + 4.07Y + 4.13Z \geq 4.78\\
&X + 3.74Y + 3.50Z \geq 0.50\\
&X + 3.75Y + 3.52Z \geq 2.55\\
&X + 3.02Y + 2.29Z \geq 2.61\\
&X + 4.42\,Y + 4.88Z \geq 49.41\\
&X + 5.73Y + 8.21Z \geq 19.02\\
&X + 5.95Y + 8.85Z \geq 0.07\\
&X + 4.04Y + 4.08Z \geq 6.06\\
&X + 3.83Y + 3.67Z \geq 0.42\\
&X + 3.50Y + 3.05Z \geq 0.40\\
&X + 0.10Y + 0.003Z \geq 56.35\\
&X + 2.61Y + 1.71Z \geq 9.63
\end{aligned}$$

$X + 0.10Y + 0.003Z \geq 56.35$

$$
\begin{aligned}
&X + 8.18Y + 16.71Z \geq 9.91\\
&X + 5.47Y + 7.48Z \geq 0.02\\
&X + 4.30Y + 4.61Z \geq 0.11\\
&X + 4.20Y + 4.40Z \geq 3.79\\
&X + 3.12Y + 2.43Z \geq 1.46\\
&X + 3.17Y + 2.52Z \geq 10.64\\
&X + 7.44Y + 13.82Z \geq 0.02\\
&X + 5.86Y + 8.58Z \geq 0.19\\
&X + 4.38Y + 4.79Z \geq 1.87\\
&X + 4.09Y + 4.18Z \geq 0.12\\
&X + 3.84Y + 3.68Z \geq 4.36\\
&X + 3.37Y + 2.84Z \geq 8.64\\
&X + 7.28Y + 13.26Z \geq 0.73\\
&X + 5.89Y + 8.67Z \geq 0.41\\
&X + 4.54Y + 5.15Z \geq 1.41\\
&X + 4.07Y + 4.13Z \geq 0.00713
\end{aligned} \qquad (19)
$$

Where the last constraint was varied by selecting respective random combinations (summarized in Table 3.) of the controlling factor "**X**".

From Table 3 the orthogonal condition of $\{ x_{19}, x_1 \}$ should be chosen, because this choice leads to the smallest value of J (objective function). Table 3 also shows that applying fuzzy regression techniques, for solving a regression problem accounting for variations and unclear defined parameters, to all variable values (X, Y, Z, and J or optimal value) satisfies the "Positive Definite Property". The "Positive Definite Property" is defined as:

$$
\begin{aligned}
&X_i > 0,\\
&Y_i > 0,\\
&X_i\, Y_i - Z_i^2 > 0 \qquad \text{for all } i
\end{aligned} \qquad (20)
$$

As a result, combination $\{ x_{19}, x_1 \}$ thus satisfies the "Positive Definite Property" where D_A is defined as:

$$D_A^* = 58.26(15.89) - (-19.55)^2 > 0 \qquad (21)$$

Although, the fuzzy regression yields to results that satisfy the assumptions defined by its theory, when compared to other methods applied in past experiments values for the objective function and covariance matrix were always significantly lower than the ones obtained with fuzzy regression. Then, it could be inferred that although our sample was chosen to be small (since fuzzy regression main theory relies on having reduced data sets) the randomness of the behavior of the factors (age and fume) is influencing the shelf-life of the gun propellants. Also, from Table 3 no pattern behavior seems to define the objective function nor its variable components.

To find the tendency and distribution for modeling the shelf-life of gun propellants following the fuzzy regression, 90 combinations repeating each variable exactly five times were randomly paired (refer to Table 4.). The linear programming problem defined before by Eq. (19) was solved with "LINDO" for each of the combinations by altering the last quality constraint to be representative of each individual combination from the group chosen. Since the 90 randomly selected combinations were forced to include exactly five times the value of each variable, for the goodness of establishing a patterned behavior to define the distribution the five values found for each combination were averaged.

Plots for each of the variable averages and standard deviations were done (refer to Figures 1-8). From Figures 1-4 a trend behavior could be observed in which maximum values occur at variables X4, X10, and X16. The behavior of the function seems to resemble an exponential form that tends to fade out as the number of variables is augmented. It needs to be pointed out that variation from the traditional distribution form is expected since we are dealing with fuzzy data sets. Therefore, an exact behavioral pattern distribution is not expected to be found.

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Standard deviation plots (refer to Figures 5-8) were sketched following the same procedure as the average plots per variable measured (refer to Figures 1-4). Results were almost identical, in which a trend was observed in all plots where maximum values were obtained at variables X4, X10, and X16. The plots then followed a stabilized fading pattern in which minimal activity was detected. It also seems that the overall cycle behavior was repeated starting from variable X35. Again, the behavior that Figures 5-8 seem to resemble is one of the exponential form with variations due to the uncertainty of our parameters.

CONCLUSIONS

The overall objective of this research was to introduce a new concept to model real life problems where data is not clearly ranged and/or defined, and apply this concept to model the shelf-life of gun propellants of the U.S. Navy.

From this research it could be concluded that although fuzzy regression thus take into account variability factors of associated uncertainties, sample size, etc. it does not clearly define or models a trend behavior which would make an accurate prediction of the shelf-life of the gun propellants. It was observed that fuzzy regression should not be implemented when the quality of the data is bad, that is when data, as it is in our case, presents high variability and/or outliners. It is therefore advisable to make use of probability conventional statistical regression analysis to predict and describe the behavior of the shelf-life of gun propellant ignoring extreme outliner points in the data set. In order to account for the possible non linearity relationship of the variables affecting the life of the gun propellants and the variation found from lot to lot, several equal samples could be modeled using both traditional statistical regression analysis as well as fuzzy regression analysis. Results should then be compared and commonality of results analyzed, from which conclusions of the distribution behavior should be drawn from that basis. By using an averaged of both results we can therefore account for the appropriateness of the regression model and its sample size.

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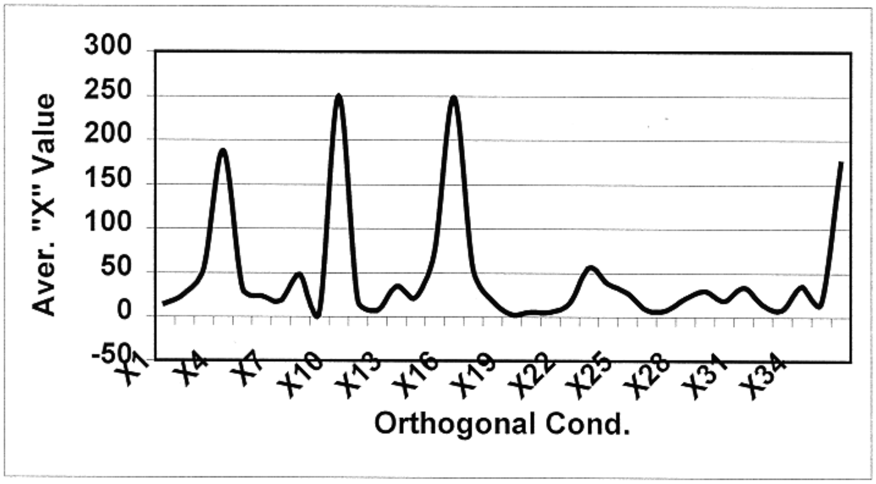

Figure 1. Plot of calculated averages of the "X" variable values.

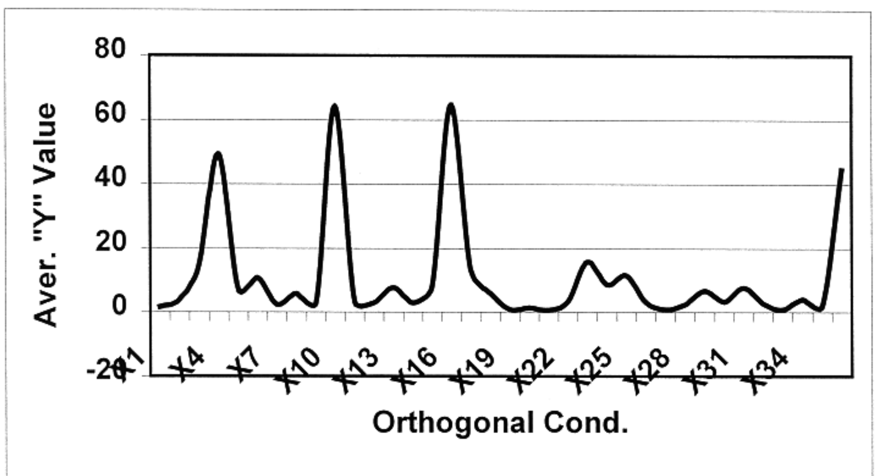

Figure 2. Plot of calculated averages of the "Y" variable values.

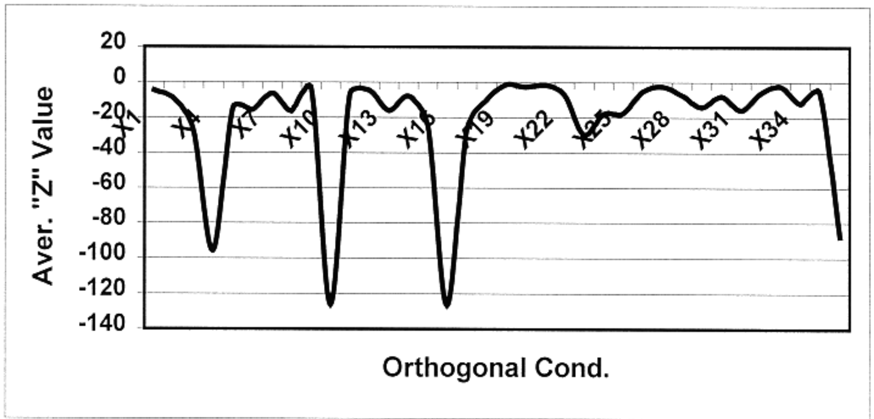

Figure 3. Plot of calculated averages of the covariance values of the function per variable.

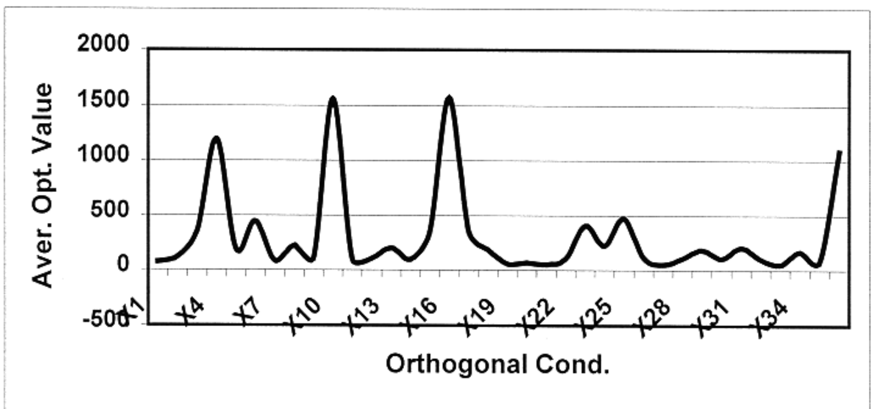

Figure 4. Plot of calculated averages of the optimal values of the function per variable.

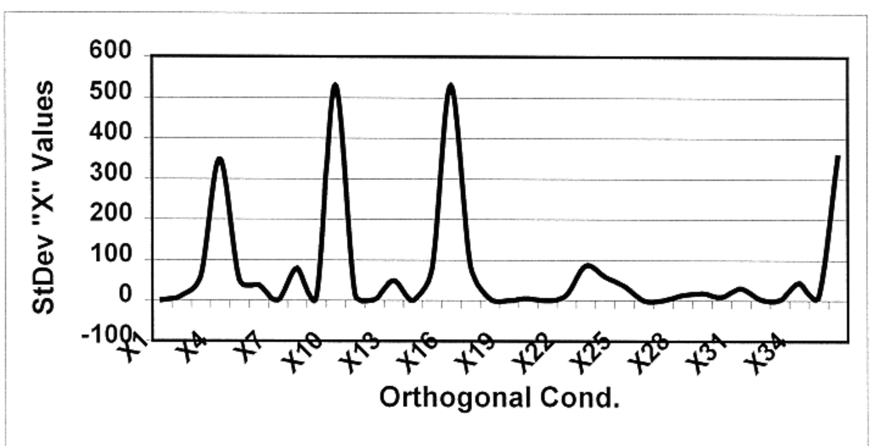

Figure 5. Plot of calculated standard deviation of the "X" variable values.

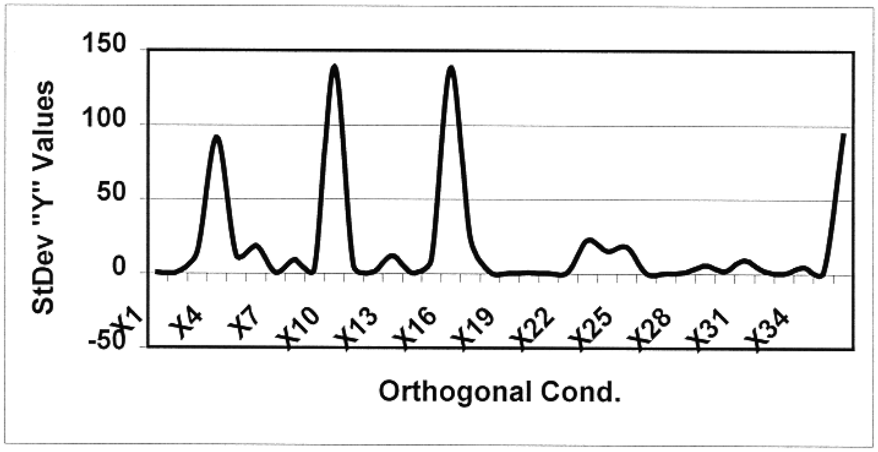

Figure 6. Plot of calculated standard deviation of the "Y" variable values.

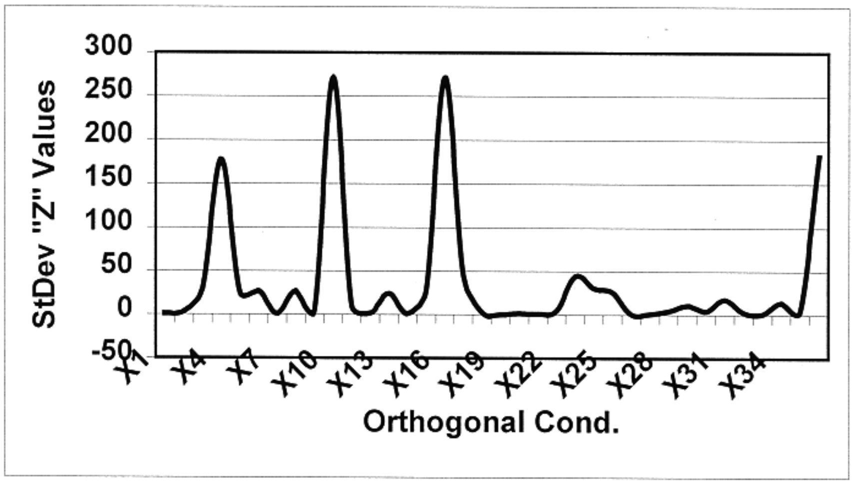

Figure 7. Plot of calculated standard deviation of the covariance per variable.

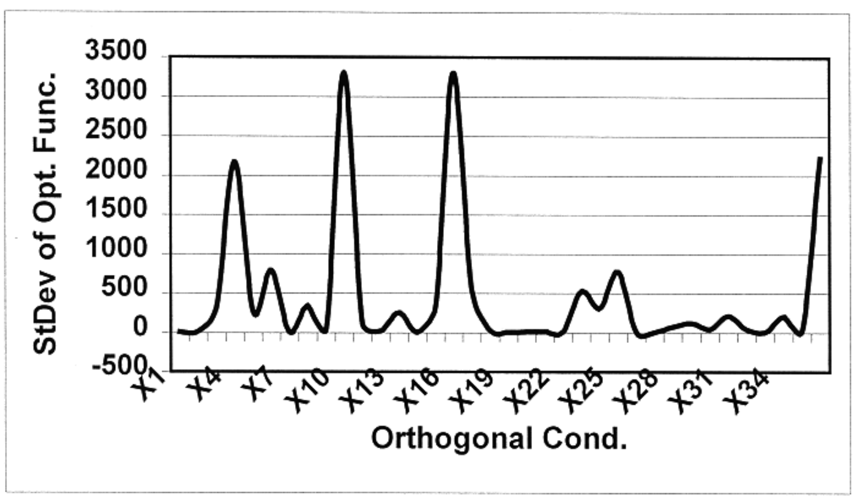

Figure 8. Plot of calculated standard deviation of the optimal function.

Table 1. Possibility vs. Probability Theories

Possibility distribution $\prod(x)$	Probability distribution $p(x)$
(Expert Knowledge)	(Frequency)
$\prod(A) = \max x\ \mu_A(x) * \prod(x)$	$P(A) = \int X_A(x)p(x)dx$
(μ_A: a fuzzy event)	(X_A: a crisp event)
Marginal possibility	Marginal probability
$\prod(x) = \max y\ \prod(x,y)$	$p(x) = \int p(x,y)\ dy$
Conditional possibility	Conditional probability
$\prod(x\|y) = k\prod(x,y)$	$p(x\|y) = p(x,y)/p(y)$
$(\max x\ \prod(x\|y) = 1)$	$(\int p(x\|y)dx = 1)$

Table 2. Observations of Age and Fume (Input – Output Data)

OBS	*AGE*	*FUME*
1	5.0247	3.06027
2	8.0849	2.74521
3	10.8274	2.03014
4	12.8548	1.88493
5	14.7397	1.94521
6	17.0904	1.58356
7	4.8521	2.95069
8	7.8027	2.85753
9	10.6575	2.03288
10	12.6877	1.87123
11	14.5589	1.87671
12	16.4329	1.51233
13	17.4329	2.20822
14	4.6384	2.86575
15	7.5041	2.97534
16	10.4767	2.01918
17	12.4932	1.91507
18	14.4082	1.74521
19	16.1507	0.05205
20	18.6658	1.30685
21	4.7836	4.08767
22	8.8685	2.73425
23	11.6000	2.14795
24	13.7479	2.09863
25	15.8438	1.55890
26	17.4027	1.58630
27	3.8932	3.71781
28	7.6082	2.92877
29	10.5370	2.18904
30	12.7233	2.04384
31	14.7671	1.91781
32	16.6822	1.68493
33	3.7041	3.64110
34	7.3425	2.94521
35	10.2877	2.26849
36	12.5534	2.03288

Table 3. Random Combinations of X & Results Obtained Applying Fuzzy Regression Analysis

Orthogonal Condition	Obtained "X" Value	Obtained "Z" Value	Obtained "Y" Value	Optimal Value of Function
x_1, x_3	728.81	-253.92	90.76	3505.20
x_{36}, x_{25}	531.63	-303.81	176.36	5686.84
x_3, x_3	967.27	-532.11	293.86	8215.20
x_{25}, x_{17}	926.19	-510.58	282.78	7976.75
x_{13}, x_{33}	369.17	-110.08	34.18	2386.65
x_6, x_{28}	793.53	-279.81	100.95	3706.49
x_{19}, x_1	58.26	-19.55	15.89	2153.84
x_{33}, x_2	427.12	-118.71	33.15	2864.33
x_{23}, x_{35}	26845.10	-11517.99	4941.36	102501.69
x_{10}, x_5	4778.78	-2587.71	1402.83	37314.32

Table 4. Results from Solving the LP Problem with Random Combination Pairs

Orthogonal Condition	*Obtained "X" Value*	*Obtained "Y" Value*	*Obtained "Z" Value*	*Obtained Value Of Function*
X1, X11	14.7	1.68	-4.93	70.87
X1, X16	15.21	1.76	-5.14	72.44
X1, X20	13.68	1.52	-4.52	67.68
X1, X13	16.29	1.93	-5.57	75.82
X1, X32	14.23	1.61	-4.75	69.41
X2, X17	29.62	4.03	-10.9	117.26
X2, X26	7.47	2.87	-4.61	104.12
X2, X29	31.16	4.27	-11.52	122.08
X2, X35	31.81	4.37	-11.78	124.1
X2, X36	30.18	4.12	-11.12	119.02
X3, X18	19.48	6.05	-10.83	172.32
X3, X24	142.77	35.01	-70.71	785.08
X3, X26	8.7	3.21	-5.27	111.53
X3, X31	89.49	23.83	-46.17	575.03
X3, X32	13.81	4.59	-7.95	141.2
X4, X21	5.86	0.516	-1.65	48.8
X4, X23	60.65	16.3	-31.4	401.59
X4, X26	10.67	3.74	-6.3	122.98
X4, X29	52.42	13.76	-26.81	335.55
X4, X36	808.92	211.52	-413.35	5052.62
X5, X12	6.72	2.65	-4.2	99.46
X5, X13	121.18	28.74	-58.9	649
X5, X14	21.87	2.81	-7.8	93.18
X5, X19	3.33	1.1	-1.72	64.56
X5, X21	5.99	0.53	-1.7	49.05
X6, X17	9.82	3.51	-5.85	118.01
X6, X22	7.39	2.85	-4.56	103.62
X6, X24	4.39	1.98	-2.93	84.86
X6, X25	91.82	43.41	-63.32	1851.72
X6, X27	6.72	0.61	-1.95	50.51
X7, X11	17.98	2.2	-6.24	81.06
X7, X25	17.03	2.05	-5.87	78.12
X7, X31	18.14	2.22	-6.31	81.59
X7, X35	20.69	2.62	-7.33	89.5
X7, 36	18.73	2.31	-6.54	83.39
X8, X13	24.24	3.18	-8.75	100.55
X8, X15	187.68	21.63	-63.71	821.77
X8, X20	3.37	1.43	-2.09	72.22
X8, X27	10.34	1.01	-3.21	57.72
X8, X28	14.82	1.59	-4.86	68.54
X9, X23	3.85	5.86	-7.04	186.36
X9, X25	7.64	2.92	-4.7	105.2
X9, X32	13.79	4.58	-7.93	141.06
X9, X33	7.99	0.752	-2.39	53.04
X9, X36	-8.37	0.951	1.09	68.63
X10, X12	7.24	2.8	-4.49	102.71
X10, X14	21.51	2.75	-7.66	92.06
X10, X15	17.11	2.06	-5.9	78.37
X10, X16	1196.6	312.66	-611.23	7462.28
X10, X21	5.83	0.513	-1.64	48.74
X11, X27	7.26	0.671	-2.13	51.58

Table 4. Results from Solving the LP Problem with Random Combination Pairs (Continued)

X11, X29	43.14	11.74	-22.48	293.16
X11, X33	7.66	0.716	-2.27	52.39
X12, X17	6.9	2.71	-4.3	100.58
X12, X18	11.02	4.16	-6.76	146.69
X12, X22	5.49	2.3	-3.53	91.72
X13, X20	3.38	1.57	-2.26	75.8
X13, X26	8.01	3.03	-4.9	107.52
X14, X30	22.46	2.9	-8.04	95
X14, X31	21.73	2.79	-7.74	92.75
X14, X36	24.07	3.15	-8.68	100.01
X15, X19	3.27	0.56	-1.11	51.55
X15, X28	45.78	4.27	-14.06	236.68
X15, X34	113.74	12.06	-37.06	535.82
X16, X26	8.78	3.23	-5.31	111.96
X16, X32	13.91	4.62	-8	141.77
X16, X33	7.96	0.749	-2.38	52.98
X17, X23	205.56	54.1	-105.37	1302.32
X17, X34	18.33	2.25	-6.39	82.17
X18, X25	17.08	6.39	-10.43	221.23
X18, X30	19.24	5.98	-10.71	171.01
X18, X31	24.09	7.22	-13.18	197.49
X19, X24	3.32	0.989	-1.59	61.8
X19, X30	3.32	1.03	-1.64	62.74
X19, X35	3.31	0.88	-1.47	59.18
X20, X29	3.39	1.58	-2.26	75.9
X20, X34	6.47	0.584	-1.86	50.01
X21, X23	3.37	1.4	-2.06	71.66
X21, X35	6.81	0.621	-1.98	50.68
X22, X24	31.47	4.32	-11.64	123.03
X22, X25	6.66	2.64	-4.17	99.07
X22, X30	31.18	4.27	-11.52	122.12
X23, X27	7.87	0.739	-2.35	52.8
X24, X27	7.75	0.725	-2.31	52.56
X28, X30	19.64	2.46	-6.91	86.22
X28, X32	18.12	2.22	-6.3	1.52
X28, X33	11.14	1.09	-3.46	60.65
X29, X34	20.2	2.55	-7.13	87.97
X31, X34	18.35	2.25	-6.39	82.22
X32, X33	7.6	0.676	-2.15	51.67
X32, X34	17.53	2.13	-6.07	79.67
X34, X35	20.91	2.66	-7.42	90.18

Table 5. Calculated Averages per Random Combination

Orthogonal Condition	*Average "X" Value*	*Average "Y" Value*	*Average "Z" Value*	*Average Opt. Value*
X1	14.822	1.7	-4.982	71.244
X2	26.048	3.932	-9.986	117.316
X3	54.85	14.538	-28.186	357.032
X4	187.704	49.1672	-95.902	1192.308
X5	31.818	7.166	-14.864	191.05
X6	24.028	10.7472	-15.722	441.744
X7	18.514	2.28	-6.458	82.732
X8	48.09	5.768	-16.524	224.16
X9	4.98	3.0126	-4.194	110.858
X10	249.658	64.1566	-126.184	1556.832
X11	18.148	3.4014	-6.752	103.702
X12	7.474	2.924	-4.656	108.232
X13	34.62	7.69	-16.076	201.738
X14	22.328	2.88	-7.984	94.6
X15	73.516	8.116	-24.368	344.838
X16	248.492	64.6038	-126.412	1568.286
X17	54.046	13.32	-26.562	344.068
X18	18.182	5.96	-10.382	181.748
X19	3.31	0.9118	-1.506	59.966
X20	6.058	1.3368	-2.598	68.322
X21	5.572	0.716	-1.606	53.786
X22	16.438	3.276	-7.084	107.912
X23	56.26	15.6798	-29.644	402.946
X24	37.94	8.6048	-17.836	221.466
X25	28.046	11.482	-17.698	471.068
X26	8.726	3.216	-5.278	111.622
X27	7.988	0.751	-2.39	53.034
X28	21.9	2.326	-7.118	106.722
X29	30.076	6.78	-14.04	182.932
X30	19.168	3.328	-7.764	107.418
X31	34.36	7.662	-15.958	205.816
X32	14.19	2.8452	-6.09	99.138
X33	8.47	0.7966	-2.53	54.146
X34	35.77	3.9968	-11.908	168.73
X35	16.706	2.2302	-5.996	82.728
X36	174.706	44.4102	-87.72	1084.734

X33 8.47 0.7966 -2.53 54.146

Table 6. Calculated Standard Deviations per Random Combination

Orthogonal Condition	*StDev "X" Values*	*StDev "Y" Values*	*StDev "Z" Values*	*StDev of Opt. Func.*
X1	0.99688	0.16	0.4	3.11
X2	10.42	0.61	3.02	7.84
X3	59.15	14.18	29.02	304.85
X4	348.12	90.99	177.92	2162.89
X5	50.48	12.1	24.74	256.83
X6	37.95	18.44	26.55	788.61
X7	1.36	0.21	0.54	4.23
X8	78.4	8.91	26.5	334.44
X9	8.27	2.23	3.66	54.25
X10	529.4	138.92	271.16	3301.31
X11	14.71	4.71	8.88	106.24
X12	2.09	0.72	1.23	21.9
X13	49.04	11.79	24.05	250.44
X14	1.04	0.16	0.41	3.21
X15	76.71	8.76	25.97	328.88
X16	530.01	138.68	271.03	3295.02
X17	85.16	22.81	44.12	535.87
X18	4.75	1.12	2.31	28.46
X19	0.02	0.21	0.24	5.09
X20	4.47	0.42	1.09	10.77
X21	1.29	0.38	0.56	10.02
X22	13.61	0.95	4.12	14.05
X23	86.88	22.36	44.03	521.57
X24	59.72	14.83	29.84	316.24
X25	35.99	17.93	25.62	773.83
X26	1.21	0.33	0.64	7.11
X27	1.39	0.15	0.49	2.77
X28	13.74	1.21	4.1	73.36
X29	19.26	5.58	10.33	122.09
X30	10.08	1.88	3.91	41.41
X31	30.92	9.28	17.12	212.06
X32	4.19	1.72	2.38	40.37
X33	1.5	0.17	0.53	3.68
X34	43.97	4.59	14.24	205.84
X35	11.61	1.53	4.3	29.13
X36	354.84	93.43	182.1	2218.2

LIST OF FIGURES

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