Bootstrapping a Stochastic Process: Time-Indexed Risk Profile Analysis of an Index Fund[1]

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Abstract. It is demonstrated how resampling can be used to obtain a risk profile analysis of an index portfolio. A variant of the partially privatized Social Security System concept is examined using a nonparametric analysis.

Introduction The use of resampling techniques for market forecasting has not proved fruitful. Indeed, the forecasting of the future value of a portfolio by any technique has defied the experts. In this paper, we take a new tack. Instead of forecasting the value of a portfolio at a future time, we forecast the entire stochastic process characterizing the risk profile of the portfolio.

At any given time, there will be this or that investment fund which is performing well above the average of the United States stock market. Some of these, such as Warren Buffett's Berkshire-Hathaway have outperformed the overall market for many years. But that is unusual.

The creator of the Vanguard S&P 500 fund, John C. Bogle [1] has long noted that investment funds tend not to outperform the weighted average of the overall market. Their stock selections, if better than the market, tend not to hold up over time. By simple chance, some funds at any given time will appear to perform wondrously well. Over time, these outperforming funds fade like flowers in winter.

The large management fees required by fund managers put them at a disadvantage relative to the performance of funds based simply on the broad market index funds. Index funds require management fees in the 0.2% per year range.

Now, if it were possible to find some funds which consistently (absent management fees) underperformed the market, that would be as useful as discovering a good fund, for then we could find what the investment policy of the bad fund was and bet against it. Unfortunately, we do not seem to have found any such funds.

Discussion

If there is no magic forecasting device to give good predictions of stock prices, then the investor still has two weapons at his disposal. One weapon is that of diversification among a number of securities. If we have ten stocks, each with the same growth rate and each with the same volatility, dividing our investment among the ten stocks rather than putting all our investment in any one of them is almost a "free lunch." Of course, the lunch is not entirely free. Such diversification should save us from losing everything in an Enron but it might kill our hopes of becoming a Microsoft millionaire (as many of Microsoft's secretaries, who had retirement plans,

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not unlike those of Enron's secretaries, became). Diversification of this sort has been used for a long time (in the nineteenth century many farmers planted corn as well as wheat in the event that hail storms zapped the more profitable wheat).

But in a bear market, the overwhelming majority of stocks decline in value. We have treated this elsewhere [2], [3], and [4] by adding Poisson bear jumps to the Gaussian walk part of a model of stock performance. Just as an extended drought will zap both corn and wheat, a bear market will hurt stocks generally. (An old politically incorrect adage of Wallstreet is "When the paddy wagon comes, good girls are arrested as well as the bad.") What other variable can we use for "diversification"? The answer is **time**.

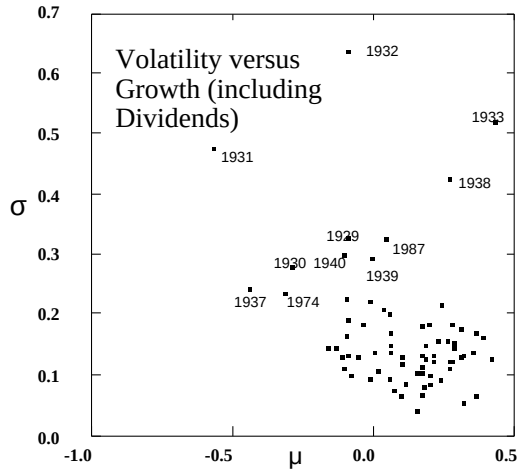

Figure 1. 75 Years of Ibbotson Index Growth and Volatility.

Investors over longer periods of time, have the advantage of the fact that in roughly 70% of the years, the index of large cap U.S. stocks rises rather than falls. And there is the further encouraging news that in over 40% of the years, the index rises by over 20%. In 30% of the years, the market rises by over 25%. And in 25% of the years, the index has risen by over 30%. Over the roughly 75 year period such records have been kept, the United States has lived through the Great Depression, the Second World War, the Cold War, Korea, Vietnam, assorted massive sociological changes, shifts toward and away from free markets, and assorted epidemics. These can all be viewed as the political/economic/sociological analogs of major "droughts." It is true that we have yet to experience Martian invasion, attacks by genetically engineered viruses or suitcase nuclear devices, or the costs of mounting the Sixth Crusade. We hope such events do not occur, but events of comparable angst have occurred to other countries of the West over the past 75 years. Poland was occupied by Russia and Germany in September of 1939, and the Russian occupation only ended (sort of) in June of 1989. It is hard to imagine a market hedge (other than taking oneself and ones money out of Poland and moving to, say, the United States) which would have saved an investor in the Warsaw Stock Exchange. And it is hard today to imagine a safe harbor for oneself or ones property in the event that the United

States falls. Past performance is not an infallible guide for predicting a risk profile and we do not claim it to be. But it is surely a guide which all should at least consider.

Table 1. Ibbotson Large Stock Index
μ (Including Dividends) and σ

Year	μ	σ	Year	μ	σ
1926	0.10993	0.11798	1964	0.15255	0.03985
1927	0.31838	0.13038	1965	0.11734	0.08560
1928	0.36193	0.16555	1966	-0.10603	0.11051
1929	-0.08796	0.32487	1967	0.21495	0.11965
1930	-0.28635	0.27484	1968	0.10490	0.12837
1931	-0.56810	0.47468	1969	-0.08883	0.13045
1932	-0.08545	0.63357	1970	0.03932	0.20436
1933	0.43172	0.51663	1971	0.35837	0.13481
1934	-0.01450	0.22225	1972	0.17379	0.06531
1935	0.38981	0.15963	1973	-0.15853	0.14316
1936	0.29207	0.14341	1974	-0.30748	0.23396
1937	-0.43124	0.23919	1975	0.31627	0.17478
1938	0.27094	0.42202	1976	0.21382	0.13136
1939	-0.00411	0.29154	1977	-0.07451	0.09550
1940	-0.10292	0.29437	1978	0.06354	0.16708
1941	-0.12319	0.14373	1979	0.16924	0.13337
1942	0.18515	0.14602	1980	0.28081	0.18309
1943	0.23032	0.15564	1981	-0.05035	0.12923
1944	0.18024	0.07712	1982	0.19400	0.18413
1945	0.31071	0.12841	1983	0.20302	0.09729
1946	-0.08414	0.18971	1984	0.06081	0.13610
1947	0.05553	0.09503	1985	0.27884	0.11859
1948	0.05354	0.19931	1986	0.16949	0.17930
1949	0.17219	0.10062	1987	0.05098	0.32354
1950	0.27543	0.10740	1988	0.15538	0.09988
1951	0.21527	0.11992	1989	0.27376	0.12009
1952	0.16865	0.11214	1990	-0.03221	0.18407
1953	-0.00995	0.09333	1991	0.26659	0.15397
1954	0.42278	0.12566	1992	0.07390	0.07315
1955	0.27429	0.12017	1993	0.09522	0.06076
1956	0.06354	0.14693	1994	0.01301	0.10559
1957	-0.11406	0.12720	1995	0.31794	0.05080
1958	0.36019	0.06137	1996	0.20758	0.08288
1959	0.11297	0.08002	1997	0.28788	0.15119
1960	0.00469	0.13557	1998	0.25138	0.21275
1961	0.23815	0.08793	1999	0.19095	0.12391
1962	-0.09135	0.20038	2000	-0.09552	0.16284
1963	0.20539	0.09662			

In the plot of μ versus σ we note some interesting years outside the apparent cluster. The two years with both high volatility and high growth are 1933 (the "Happy Days Are Here Again" optimism which characterized the start of the Roosevelt era) and 1938 (the year after the bottom of the Great Depression). Nine of the eleven outliers are depression years. Both 1974 and 1987 had significant bear epochs. With these eleven years removed, the correlation between μ and σ is $-.142$. With all 75 years left in the data base, the correlation is $+.184$.

Let us consider a portfolio with initial value of \$100,000. We pick five of the 75 index annual growths at random (with replacement), say, μ_1,μ_2,μ_3,μ_4,μ_5. Then a simulated portfolio value after the five years is given by

$$V = \$100,000 \exp(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)$$

In Figure 2, we show the picture obtained by sorting a thousand such simulations according to percentiles.

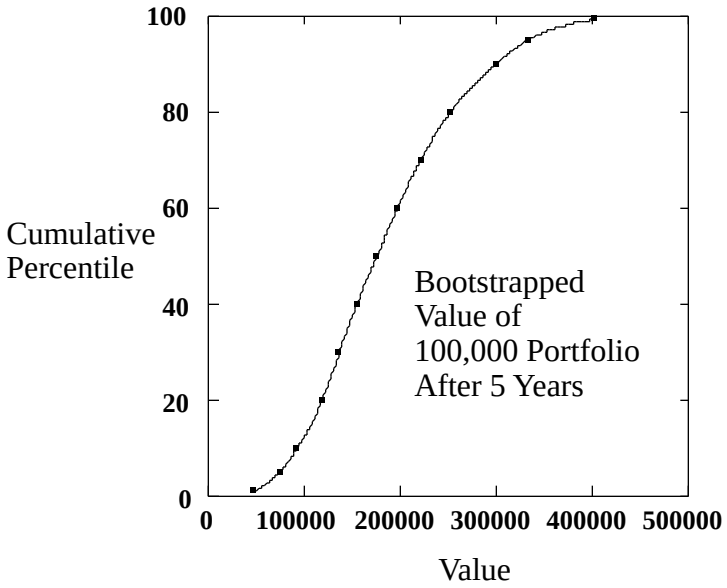

Figure 2. Distribution Function of Portfolio after 5 Years Using Resampling.

The mean value of a \$100,000 portfolio after five years is \$192,676. The median value is \$175,530 (growth rate of .1125). However, the lower ten percentile is \$92,747 (growth rate of $-.015$).

Next, we consider the same scenario except looking 20 years into the future. The results are quite optimistic. The median value is \$873,100, an annual increase of 10.8%. Even the lower ten percentile value of \$285,590 represents a growth rate of 5.2%.

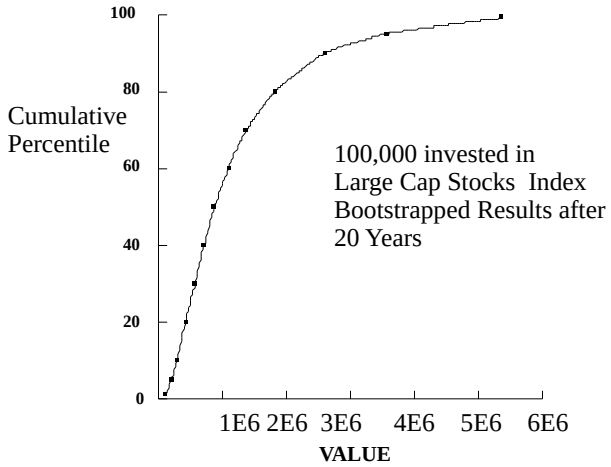

Figure 3. Distribution Function of Portfolio after 20 Years Using Resampling.

A Partially Privatized Social Security Plan President George W. Bush has suggested a partial privatization of Social Security. Under the present plan, workers and their employers together "contribute" roughly 15% of a worker's salary to the FICA fund. The worker's portion of the contribution is not tax exempt. After roughly 45 years of employment, the worker may start to draw a Social Security stipend until the time of his/her death. Part of the stipend may be subject to income tax, even though no tax exemption was given the worker while paying his FICA tax.

President Bush has suggested that a worker might elect to use a portion of his FICA setasides to invest in the stock market. Typically, it is assumed that some restrictions leaning toward fiscal conservatism will be applied. So, we will use, as an example, a contribution of $2,000 per year for each worker over a period of 45 years. We will invest the money in something like the Ibbotson Index. So, let us note in Figure 2 what the bootstrapped results look like when we take 5,000 resampled concatenations of the index, assuming that $2,000/year will be added to the fund. The results are quite promising. The mean value of such a fund is $4.84 million dollars. The median value is $2.724 million. Even the lower ten percentile is $695 thousand. The lower five percentile is $464 thousand. The lower one percentile is $225 thousand. Of course, there is the problem of inflation. Even so, we realize that the "contributions" would be indexed on inflation. And, naturally, we could index the Ibbotson index as well.

Another objection could be that the kind of massive infusion of monies into the stock market as might be caused by a partial privatization could inflate stock values in the shorter term and might lead to a collapse in the longer term. That is, persons already with substantial holdings in the stock market would receive an immediate benefit as the new funds from partial privatization poured into the market causing a

rise in stock prices. A partial privatization similar to that considered here of social security has already taken place in Sweden (instituted by the socialist government there). No apparent inflation in world wide markets has been noted. However, the number of new Swedish investors in the market is quite small compared with that which would be experienced in the United States.

It is unlikely that potential problems of market inflation will be the actual reason for not giving workers the option to put some of their FICA assets into bonds or securities. The actual reason will be the loss of control of money by the political elites. Who implicitly owns assets is not nearly as important as who controls them. The attempt to take over the health care system in the United States by the federal government during the Clinton Administration was unsuccessful, in part, because it was learned by the citizenry that over 10% of the American economy would have been transferred from the private sector to that of the state. History has shown an extreme reluctance of politicians to give up control of assets once they have achieved such control. And the Social Security program in the United States is nearly 70 years old. On the other hand, it is possible that, like the Swedish socialists, the American bureaucracy will recognize that they have little choice but to privatize an increasingly expensive and inefficient program. In any event, even if the partially privatized FICA plan is never introduced, the study in this section could be of use to a person thinking of making regular investments into a tax deferred index fund.

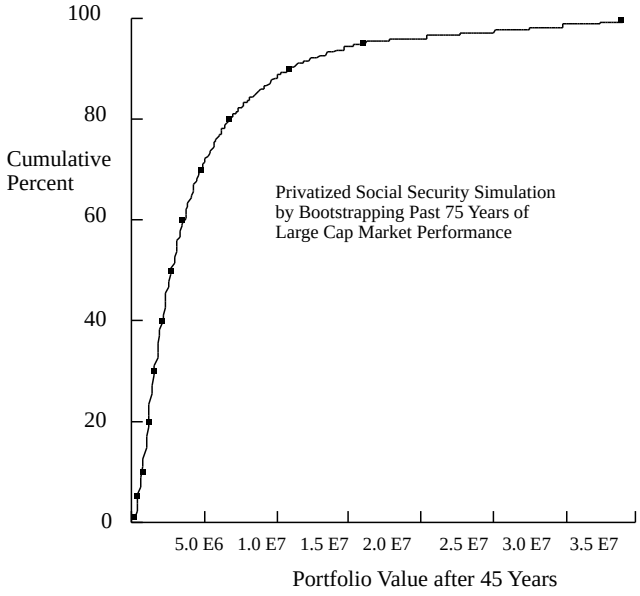

Figure 4. Resampling Based Distribution Function of Privatized Social Security Index Fund After 45 Years of Work.

Index Funds As High Interest Paying Money Market Accounts

Figure 3 and Figure 4 give some support for investing in an index fund broadly composed of large cap corporations. Note that we used an approach which has

few modeling assumptions in both these cases. We have assumed that the future increases in such an index will be similar to those in the past.

Some may object to taking single year rates from the Ibbotson history. What happens when we have long patches of decline? Might not inclusion of these in an appropriate fashion introduce more pessimism into both long term investments in index funds and the hypothetical privatized Social Security plan. In Figure 5 we show the cumulative percentiles one obtains when examining a $100,000 investment for five contiguous years starting in 1926 and going through 1996 (we are limited by 1996, since we are looking at 1996 and the four following years).

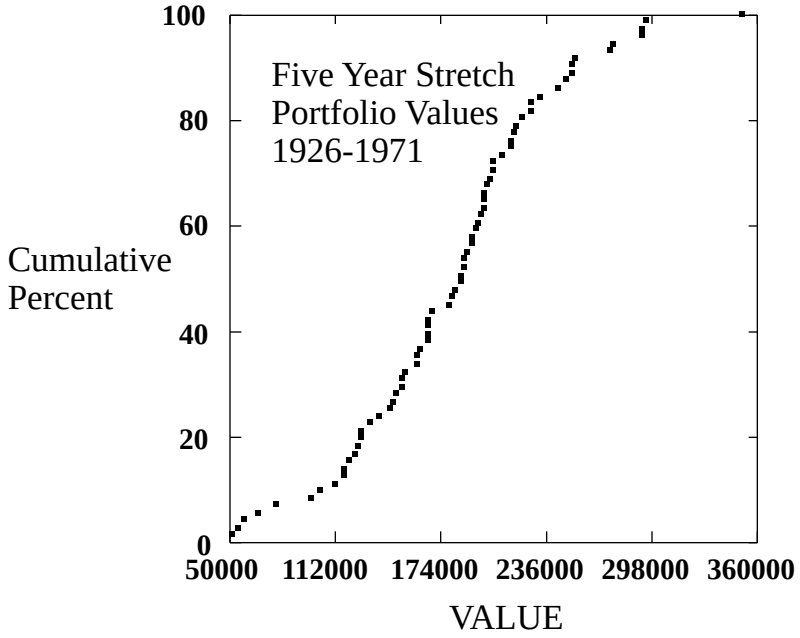

Figure 5. Resampling Based Distribution Function of $100,000 invested for Five Contiguous Years.

Now the lower ten percentile represents almost no gain at all. On the other hand, the median and mean both correspond to an annual gain (again, including dividends) of around 12%. Still, there is the troubling lower ten percentile.

Next, we carry out 10,000 resamplings of two five year stretches from the Ibbotson Index with an initial $100,000 investment in the index. This time we show a histogram of the results in Figure 6. The lower five percentile is slightly better than break even. The lower ten percentile now pays over 2.7%. The lower twenty percentile pays 4.7%. The median pays 11.3%. The mean pays 11.7%. Perhaps we can say that at ten years we really have reached the point where we can talk meaningfully about "the long term." An investor in the index fund for ten years would appear likely to be pleased with his/her end results and has little chance of awful

results. In other words, risk would appear to have been reduced to bearable levels. We note the shape of the histogram (which has been based on no distributional assumptions) has the characteristic shape of the log normal density function.

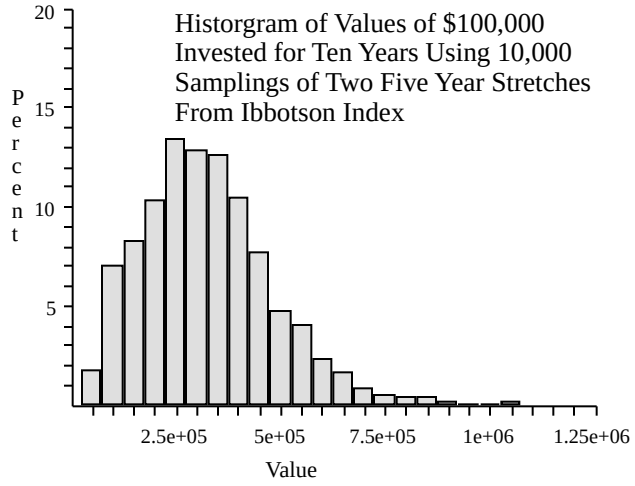

Figure 6. Resampling Based Distribution Histogram of $100,000 invested for 2 Five Contiguous Year Stretches.

Next, let us look at an index fund starting with $100,000 randomly selecting four five year stretches from the Ibbotson index of large cap stocks. We show these results in Figure 6.

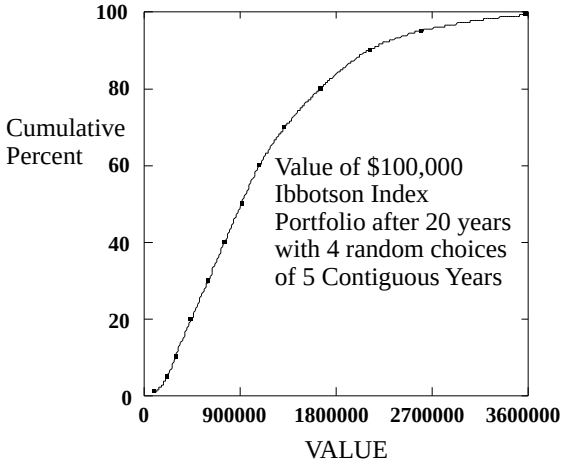

Figure 7. Resampling Based Distribution Function of Initial $100,000 Invested in Ibbotson Index for 20 Years.

The riskiness is reduced still further. The lower ten percentile of performance is a growth of 5.3%. The median growth shows over 11% annual growth and the mean over 12%.

Finally, we return to the notion of bootstrapping a 45 year investment from an annual $2,000 per year invested in the index fund. Here we pick randomly 9 five year stretches. For each year, we add $2,000 and we suppose the annual increase in the fund is the average of the five year stretch in which the year lies. We note here that the lower ten percentile is $1.32 million, the mean $4.4653 million and the median $2.5380 million.

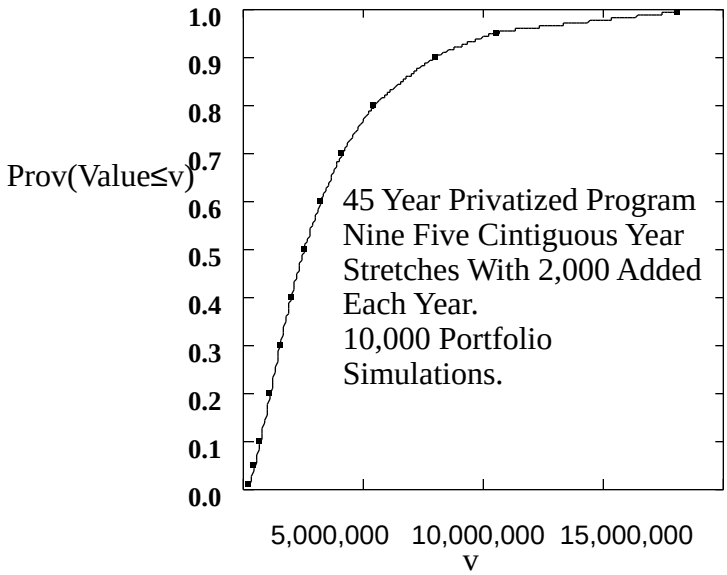

Figure 8 . Resampling Based Distribution Function of Privatized Social Security Index Fund After 45 Years of Work by Selecting Randomly 9 Five Contiguous Year Stretches from Ibbotson Index.

In any event, it would appear that if history of growth is the best guide to the future, large cap index funds appear to be very attractive. Our results show that, over the long haul, they appear to act like money market accounts paying high rates of interest (over 10%) with relatively small chance that the investor will be disappointed.

References

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