Risk Assessment: A Game Theoretic Approach

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Background

- **Game theory is a useful model for security risk assessment:**
  - **Appropriate when protecting against intelligent and adaptable adversaries**
  - **Recognizes that defensive strategies must take attacker behavior into account**
  - **Can identify qualitative properties of optimal solutions (e.g., randomization)**

Background…

- **Game theory is only beginning to be used in security risk assessment**
- **Military analogies (Schneier):**
  - **“The defender has to defend against every possible attack”**
  - **“The attacker…only has to choose one attack, and he can concentrate his forces on that one attack”**

Background…

- **Most applications are still exploratory:**
  - **Illustrative applications to the choice of attack and defense strategies (Cohen)**
  - **Experiments demonstrating relevance of game theory to information warfare (Burke)**
  - **Application of game theory to financial institution risks (Chaturvedi et al., Gupta)**
  - **Importance of perverse incentives (Anderson)**

Outline of this work

- **Games between attackers and defenders:**
  - **Simple series/parallel systems**
  - **Components with inherent values, and also a value to system function**

Overall goal

- Study optimal allocation of resources for protection of series and parallel systems against intentional attacks
- Protective investment c_i reduces the probability of successful attack against component i to $p_i(c_i)$:
  - $p_i(c_i)$ convex, decreasing, twice differentiable and invertible

Cases being considered

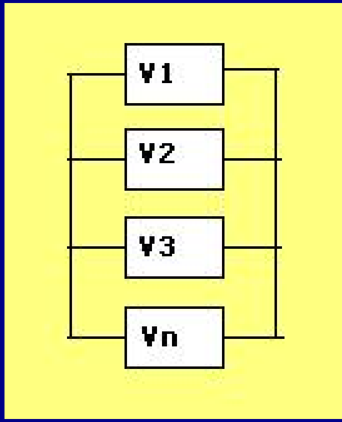
- Results to date:
  - Components in parallel
  - Components in series
  - “Additive” models
    (Components have different “values” v_i)
- In process:
  - Arbitrary series/parallel structures
    (NP-hard, may use heuristic approaches)
  - Other configurations
    (Explore merits of perimeter defense, etc.)

Assumptions

- Realistic levels of defensive investment will not deter attacks:
  - Models applicable to determined attackers
- Attacks against different components succeed or fail independently:
  - Models applicable to functionally diverse and spatially separated defenses
- Likely to apply to most serious threats against security-critical systems

Components in parallel

- Defender wishes to maximize (expected value of system) – (defense cost), or equivalently:
  - Choose c_i to minimize $\alpha\ [\Pi p_i(c_i)v + \Sigma p_i(c_i)v_i] + \Sigma c_i$
    where α is probability of an attack on the system,
    v is the value of the system functionality, and
    v_i is the inherent value of component i
- Optimum occurs when
  - $p_i'(c_i) \geq -1/\alpha[v_i + v\ \Pi_{j\neq i}\ p_j(c_j)]$, and
  - $c_i\ [\alpha\ v_i\ p_i'(c_i) + \alpha\ v\ p_i'(c_i)\ \Pi_{j\neq i}\ p_j(c_j) + 1] = 0$
- Multiple local optima are possible



Components in parallel…

- **Local optimum is unique when $p_i(c_i)$ are log convex:**
  - **Success probability decreases "faster than exponentially" in c_i**
- **This seems unlikely to be the case**

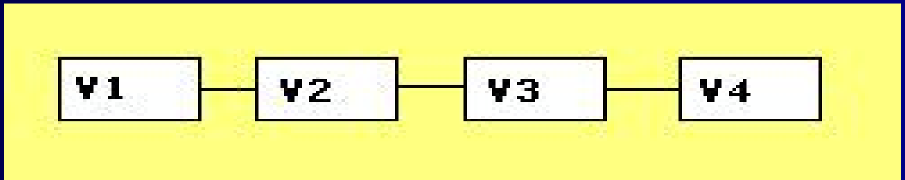
Components in parallel…

- **General insights:**
  - **Optimal defense strategy depends on the cost-effectiveness with which components can be improved**
    - **As measured by the $p_i'(c_i)$**

    and on the values of the components
    - **As measured by the v_i**
  - **Components that are too costly to defend (relative to their value) will not be hardened**

Components in series

- Can occur for many reasons:
  - Physically in series (e.g., pipelines)
  - Multiple failure modes
  - Attacker can afford only one target
  - First successful attack is much more serious (e.g., for symbolic reasons)



Components in series…

- Attacker has a **choice of targets**
- Two bounding cases:
  - Attacker has no knowledge of defensive investments
  - Attacker can obtain perfect knowledge about defensive investments at no cost

Components in series…

Perfect knowledge

- Assumption of perfect knowledge may not always be unrealistic:
  - Due to the openness of our society
- Public demands knowledge of defense
  - Even when this weakens its effectiveness!
- This increases the difficulty of defense:
  - E.g., anthrax protection

Components in series…

Perfect knowledge

- Assume attacker has only one attempt (multiple attacks are considered later)
- Attacker objective is to:
  - Choose i to maximize $[p_i(c_i)\ v_i]$
- For optimal allocation of defensive resources:
  - Defense must equalize the expected values of attacks against all targets
  - "Each of the defended targets [must] yield the same payoff to the attacker" (Dresher)

Components in series…

Perfect knowledge

- **Unlike in defending against accidents or acts of nature:**
  - **Optimal allocation does not depend on cost-effectiveness of investments!**
- **Defender is deprived of flexibility:**
  - **Must defend all targets of comparable expected value equally (regardless of cost)**

Insight

- “Investment in defensive measures,
  - unlike investment in safety measures,

  saves a lower number of lives” (Ravid)

Components in series…

Perfect knowledge

- Now, assume that the attacker can attack each component once (multiple attacks)
- Attacker objective is to:
  - Choose i to maximize
    $\alpha\, [\Sigma p_i(c_i)\, v_i + v\, \{1 - \Pi\, [1 - p_i(c_i)]\}] + \Sigma c_i$
- For optimal allocation of defensive resources:
  - Defense need not focus exclusively on components that cause highest expected damage
  - Investment in other components may pay off, if attacks against such “first-choice” targets fail
  - Optimal defense strategy again depends on the cost-effectiveness with which components can be improved

Components in series…

Perfect knowledge

- **Insights:**
  - **Properties of the optimal solution for series systems with multiple attacks are similar to those for parallel systems (e.g., multiple optima)**
  - **If one component dominates the risk, then the optimal solution with multiple attacks will be similar to that with a single attack**

Components in series…

No knowledge

- **Assume:**
  - **Attacker targets component i with constant probability q_i (regardless of defense c_i)**
  - **Attacker has only one attempt**
- **Defender objective similar to previous:**
  - **Choose $\{c_i\}$ to minimize** $\sum q_i v_i p_i(c_i) + \sum c_i$
- **Optimum occurs when $p_i'(c_i) \geq -1/(q_i v_i)$**
  - **and $c_i [q_i v_i p_i'(c_i) + 1] = 0$**
- **Expenditure c_i is increasing in $q_i v_i$**

Arbitrary Structures

- **Find defensive strategy when optimal attack strategy is NP-hard (joint work with Cox, Azaiez):**
  - **Cox's work on least cost diagnosis (1989, 1996) suggests near-optimal heuristic attack strategies**
  - **Identify optimal (or near-optimal) defenses against near-optimal attacks**
  - **Determine when heuristic attack strategies are in fact optimal**

Conclusions

- Protection of series systems from knowledgeable adversaries is a **fundamentally different** challenge:
  - Investments less cost-effective (since attacks can be deflected to other targets)
  - Defender loses flexibility to allocate resources cost-effectively
  - Importance of redundancy, secrecy (and deception) as defensive strategies

Conclusions…

- Defender should consider the success probabilities of attacks against various components:
  - Not only their inherent values
- Some high-value targets with a low probability of being successfully attacked may not merit any investment:
  - Lower-value, more vulnerable targets may merit defense
- Contrast this to the heuristic proposed by Brookings (2002):
  - Protecting only the most valuable assets