Application of
Fisher's Combined Probability Test
To the Validation of the AIM-9X Missile Model

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Arthur Fries
Institute for Defense Analyses
afries@ida.org

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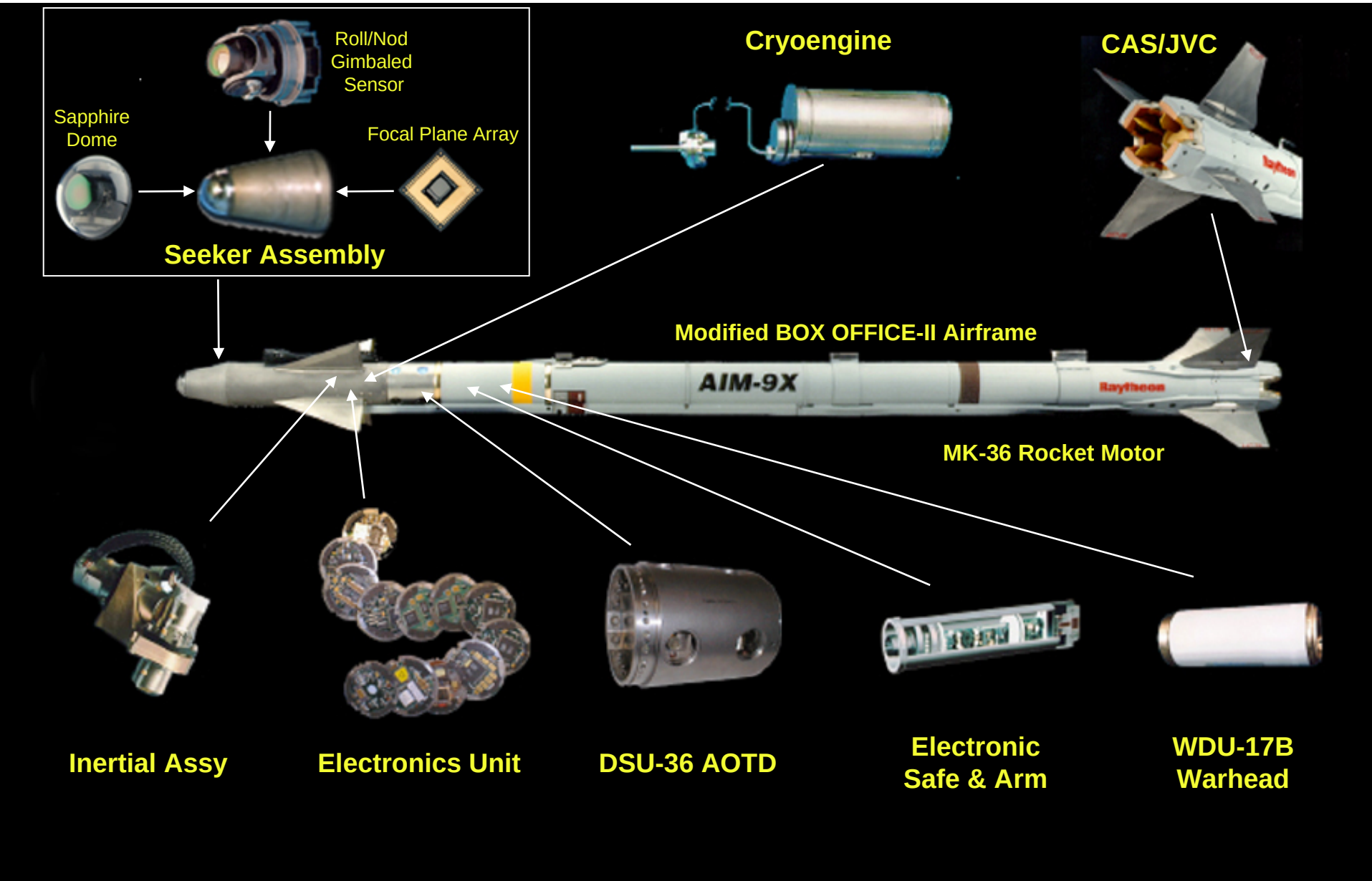
The contents of this presentation and the views therein are solely those of the author. No official endorsement by IDA, DoD, or ODOT&E is intended or should be inferred.

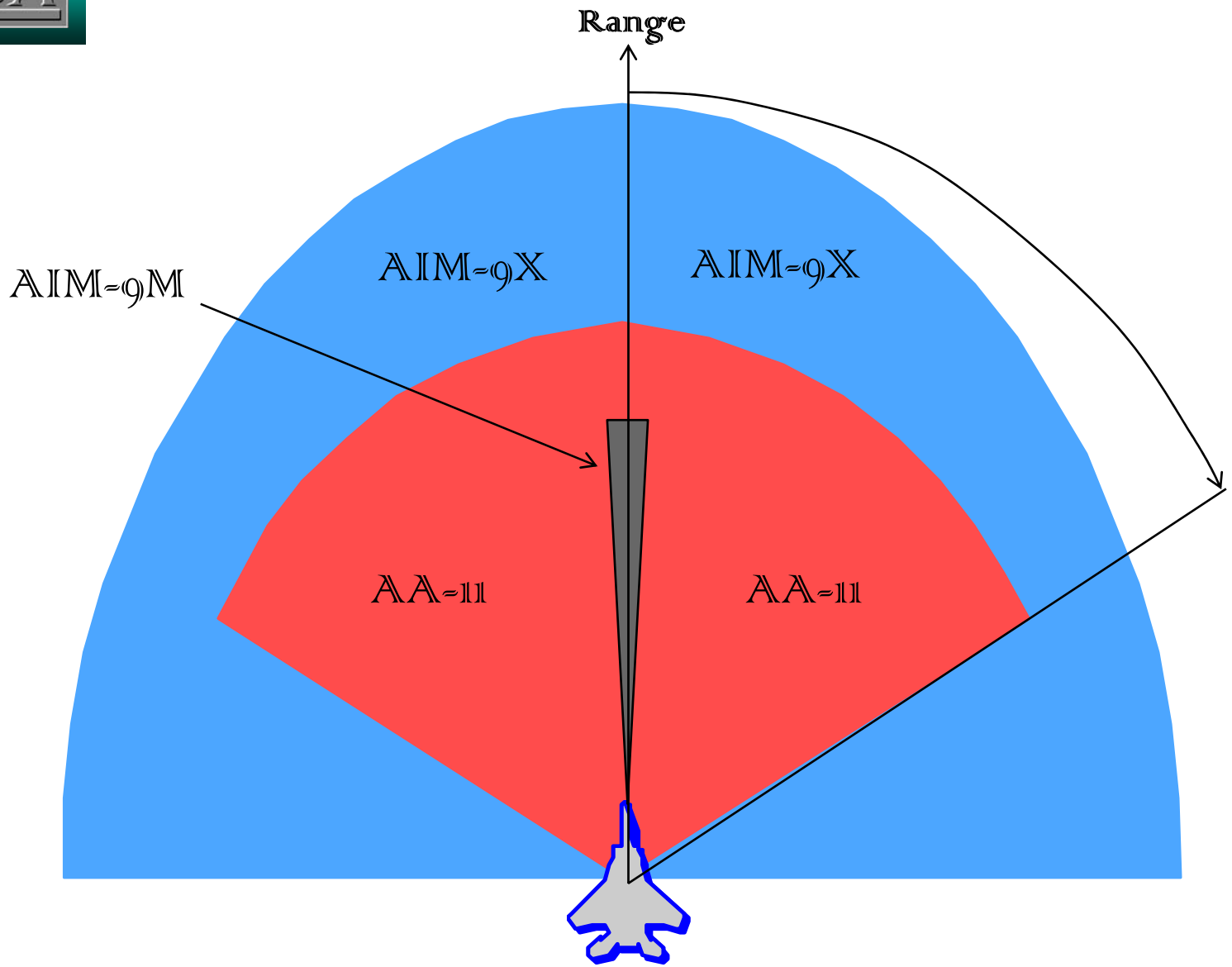
OUTLINE

- **AIM-9X**
  - System Description
  - VV&A Considerations
    - » How much to test?
    - » Where to test?
    - » How to analyze?
- **Fisher's Combined Probability Test**
  - Connection to M&S
  - Methodological considerations
    - » Accommodating measurement errors
    - » 1-sided or 2-sided testing?
    - » Other meta-analysis approaches
    - » Goodness-of-fit procedures
  - Statistical power
- **Application to AIM-9X**
- **Other Challenges**

AIM-9X: System Description



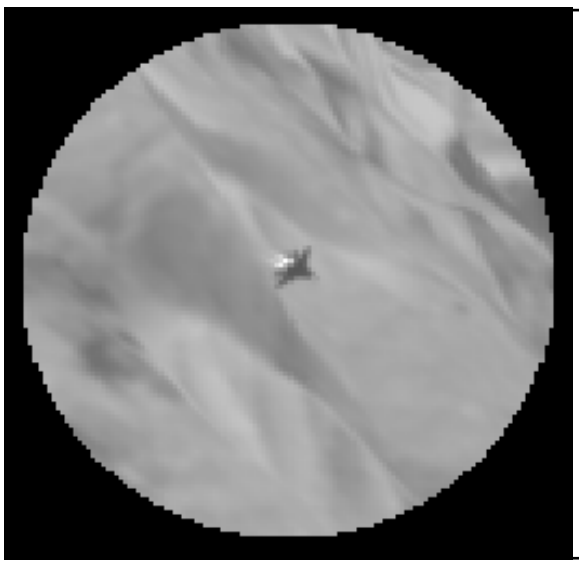
AIM-9X Launch Envelope (notional)

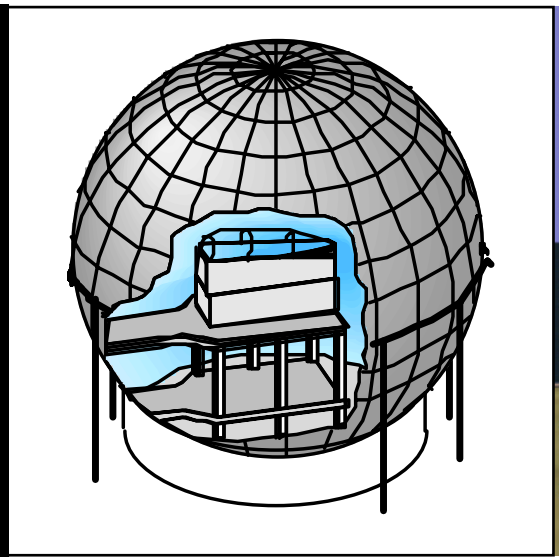


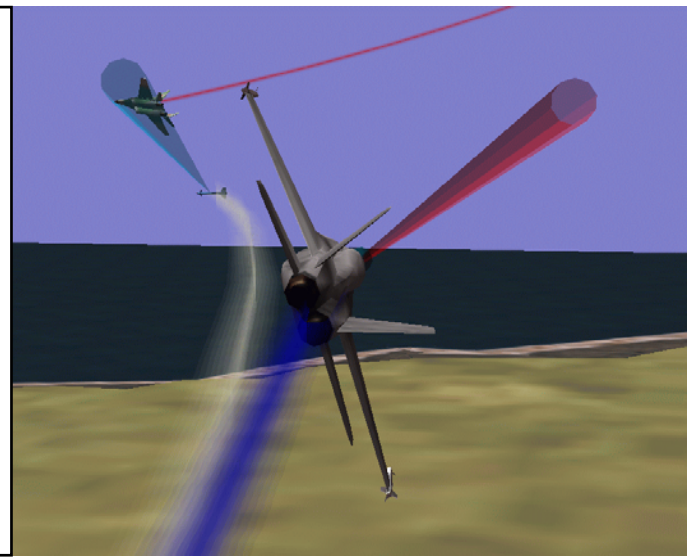
Modeling & Simulation Based Acquisition Program

- **AIM-9X Relies on Extensive Modeling & Simulation**
  - Improve the quality of flight tests
  - Reduce costs

- **Simulation Is Used to Predict All Levels of System Performance**
  - Weapon System, Missile System, Seeker, Kinematic
    - » Pk Performance Verified by Simulation Only
    - » Guided Flight Tests Used for Simulation Validation
  - Integrated Plan With Acquisition, DT, & OT Communities







Why Simulation Based Acquisition?

Simulation Based Acquisition (SBA) can lead to dramatic reductions in weapon system testing

Example: AMRAAM vs AIM-9X Full Scale Development

AMRAAM (1985-1990)	
Control Test Vehicle Flights*	29
Development Guided Flights	79
Operational Test Flights	17
Total Guided Flights	96

AIM-9X (1999-2002)	
Seperation Control Test Vehicle Flights*	22
Development Guided Flights	16
Operational Assessment Flights	5
Operational Test Flights	22
Total Guided Flights	43

Key Elements of Simulation Credibility

- **Software Accuracy**
  - Correctness of implementation
  - Accuracy of any data manipulations
    - » Unit conversions, coordinate transformations, etc.
  - Accuracy of software documentation

Measured by "verification"

- **Output validity**
  - Comparison with the "real world" (properly defined)
  - Accuracy of simulation inputs

Measured by "validation"

- **Software Stability**
  - Adequate management and control of M&S software, embedded data, and software documentation

"Configuration Management"

- **Data Accuracy**
  - Accuracy of any embedded data
    - » Physical constants, system parameters, etc.
  - Accuracy of any input data

"Data Quality"

Accreditation activities ensure robust assessments of simulation credibility in all these dimensions

WHAT IS "VALIDATION"?

- **Broad VV&A Context (AR 5-11, Management of Army Models and Simulations, 1997)**
  - "Validation is the process of determining the extent to which the M&S adequately represents the real-world from the perspectives of its intended use."

- **Limited Statistical Context**
  - Quantify consistency between observed data (real world or test) and predicted outcomes
  - Provoke detailed follow-on discussions and analyses

- **No "One-Time" Validation**
  - Same model for different purposes
  - Continuous comprehensive evaluation

AIM-9X Flight Test Matrix

	DTIIB/C				OTIIA					DTII B/C					DTIID			
	Guided Flight Sequence																	
System Simulation Aspect Validated	e1	e2	e3	e4	o1	o2	o3	o4	o5	p1	p2	p3	p4	p5	p6	p7	p8	p9
Target Signature (**L**ow, (**-**)med, **H**i)	-	-	-	-	L	-	-	-	-	-	-	H	-	-	-	-	-	-
Target Terminal Aspect (**N**ose, **T**ail, **P**lanform, **B**eam)	B	P	P	P	N	B	N	B	P	B	P	B	P	B	P	N	P	T
Closing Velocity (**L**ow, (**-**)med, **H**i)	L	-	-	-	-	-	-	-	-	-	-	-	L	-	H	-	L	L
OBA ((**-**)Low, **H**igh(vignetting))	-	-	H	H	H	-	H	-	H	-	H	H	H	-	-	-	-	-
CM (**m**idcourse,**t**erminal w/ **A**,**B**,**C**,**AC** or (**-**) none	-	-	-	mB	-	-	-	mB	mB	B	B	-	A	AC	-	mA	C	
CM - Lag**(-),** Lead (**+**), (**-**)n/a	-	-	-	+	-	-	-	+	+	+	(-)	-	(-)	+	-	(-)	+	+
Altitude (**L**ow, (**-**)**M**ed, **H**igh)	-	-	-	-	-	-	-	-	-	-	-	-	-	L	H	H	L	H
Background type: (blue sky [**bs**], **d**esert, **c**loud, **s**ea, **h**orizon)	bs	d	c	d	bs	bs	bs	d	d	d	bs	h	d	s	bs	bs	d	bs
Contrast: positive(**+**), washout(**0**), neg washout**(-)**	+	+	+	+	+	+	+	+	+	0	+	+	+	+	+	+	(-)	+
Range (**L**ow, (**-**)Med, **H**igh)	-	-	-	-	H	L	L	-	-	-	-	L	-	H	H	-	-	H
Target Maneuver ((**-**)None, **I**n-plane, **O**ut of plane)	-	-	I	I	-	I	-	O	I	-	I	-	I	O	O	I	-	-
Initialization (**R**adar, **H**elmet)	R	R	R	H	H	R	H	H	H	R	H	H	H	R	R	R	H	R
Dome heating ((-)Moderate, **H**igh)	-	-	-	-	-	-	-	-	-	-	-	-	-	H	H	H	H	-
Afterburner (**O**n, On **M**idgame, On **T**erminal, (**-**)None)	-	-	M	-	-	-	-	-	M	-	M	O	M	O	-	-	T	T
	CL	WS	WS	CL	CL	EG	EG	CL	CL	CL	CL	CL	CL	CL	EG	EG	CL	CL
	30 Jun99	1 Sep99	16 Dec99	31 Mar00	21 Apr00	23 May00	25 May00	28 Jun00	21 Jul00	12 Jun00	26 Sep00	17 Nov00	21 Dec00	5 Apr01	May-01	Aug-01	Sep-01	Oct-01

EDM DT shot

EDM OT shot

PRM DT shot (DTIIB/C)

PRM DT shot (DTIID)

PRM DT Assist

IMPLEMENTATION CONSIDERATIONS

- **How Much To Test?**
- **Where To Test?**
- **How To Analyze?**

HOW MUCH TO TEST?

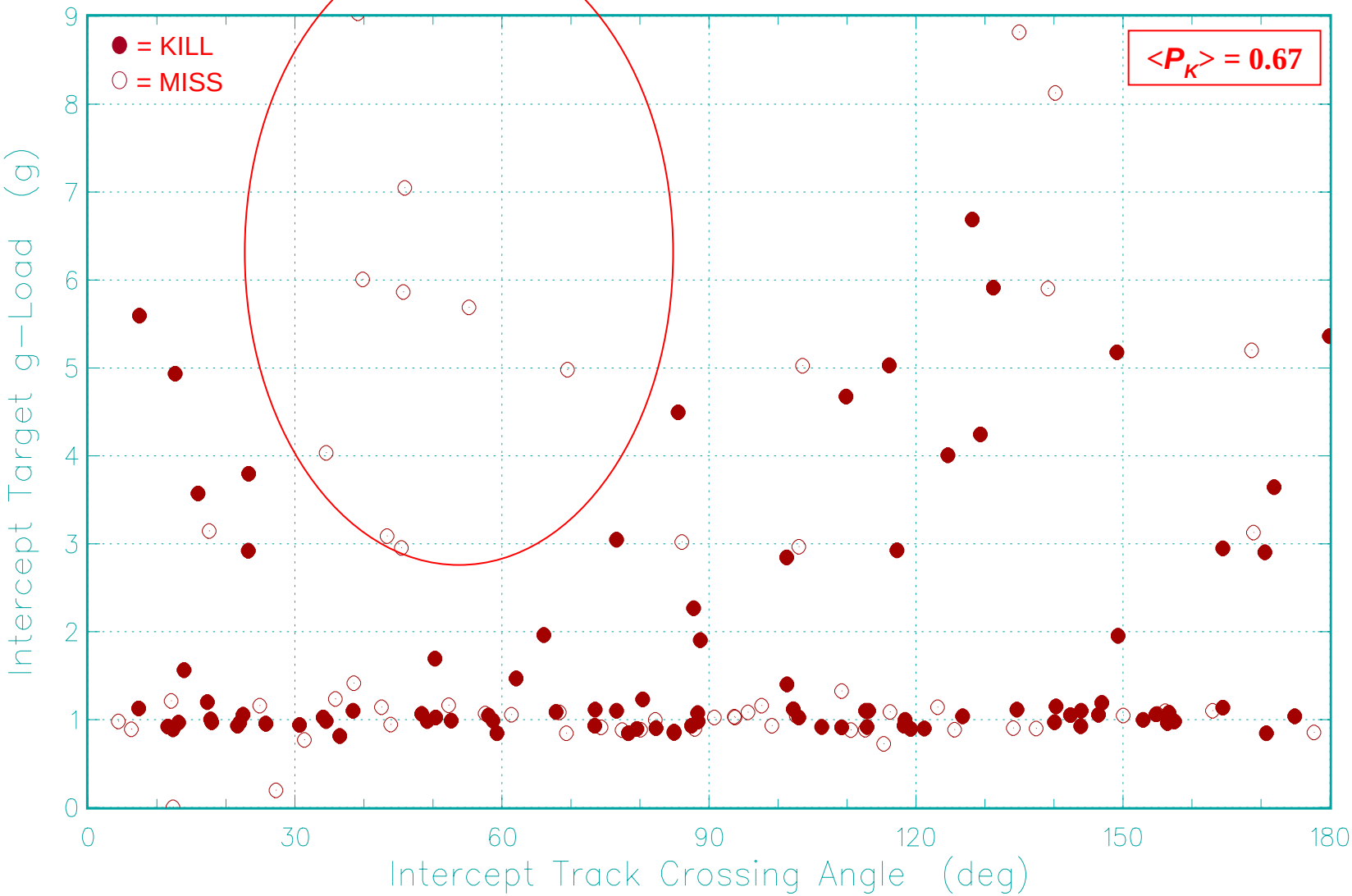
- **Whatever You Can Get**
- **20 – 40 ??**
- **Simulation-Based Investigation**

WHERE TO TEST?

- **One Point Replicated**
- **Few Points & Less Replication**
- **“Many” Points & No Replication**
  - Uniformly distributed
  - Concentrate on disparate “pockets”
    - » Based on M&S insights
    - » Based on physics insights
  - Include specific combinations
    - » Likely
    - » Extreme

M&S ALONE MAY BE INADEQUATE

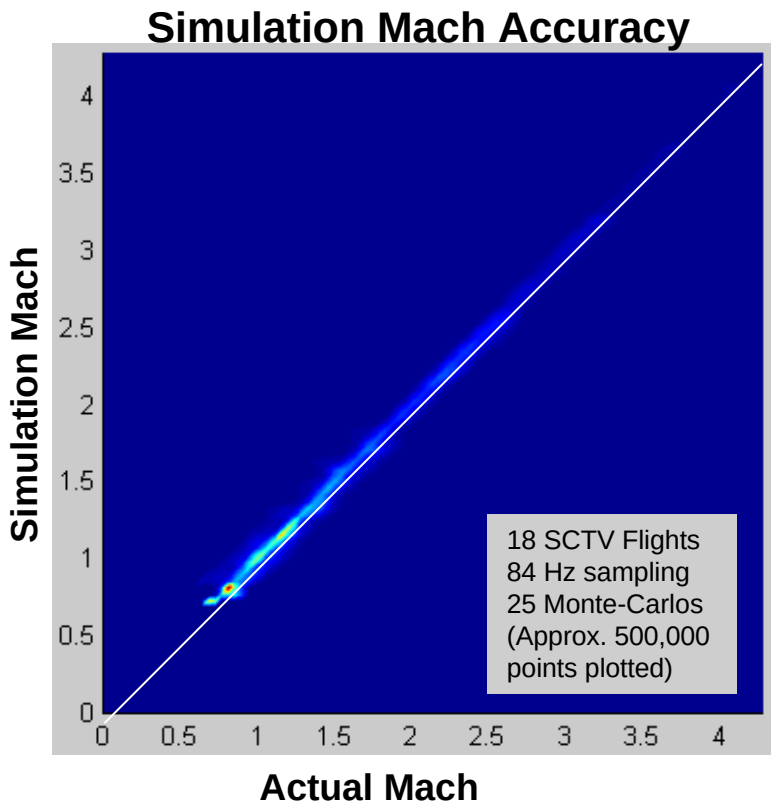
(hypothetical example, not AIM-9X)

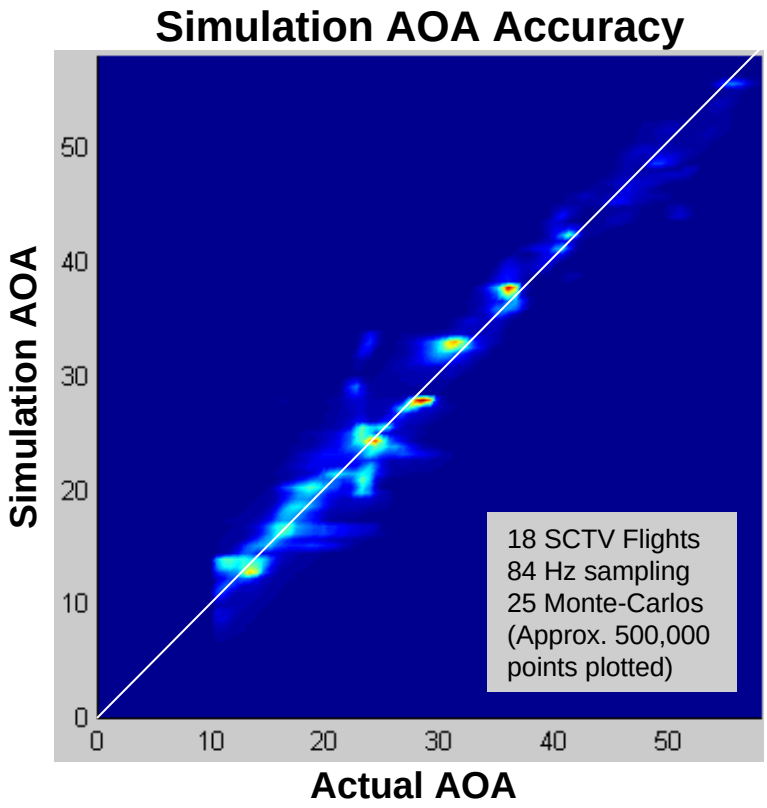


HOW TO ANALYZE?

- **Focus On End-Game Miss Distances**
  - *Old Method*: Look at the actual vs predicted miss distances and use our judgment
  - *Better Method*: Determine if actual miss distances are statistically plausible when compared to simulation predictions

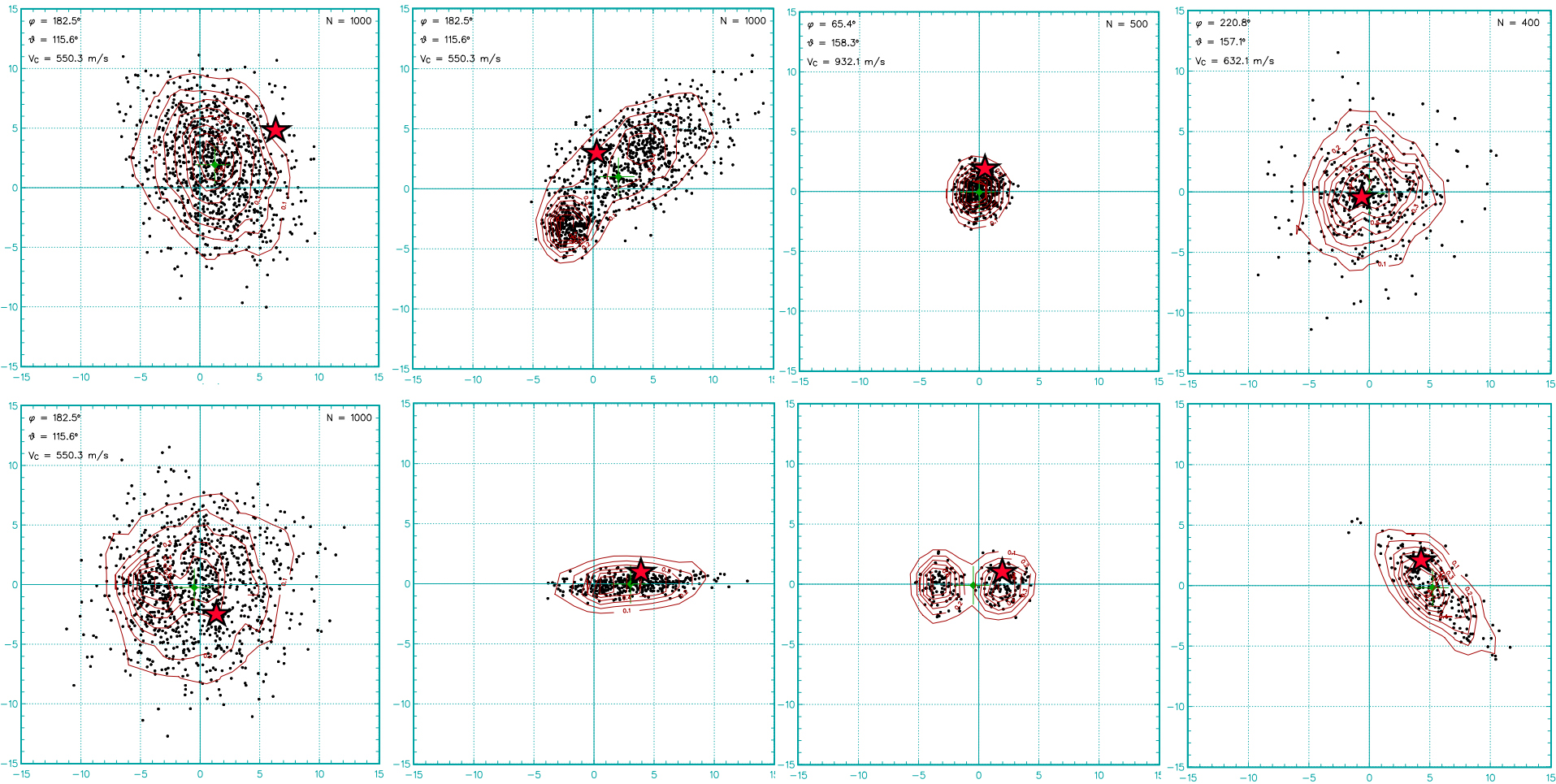
Excellent Kinematic Match





- **Actual vs simulation values of Mach and AOA are plotted for all 18 SCTV flights**
- **Excellent overall 1:1 correlation is evident**

HOW TO ANALYZE MISS DISTANCES?



FISHER'S COMBINED PROBABILITY TEST

- **Meta-Analysis Technique**
- **Combines Information From Different "Experiments"**
  - Same H_0
  - Different settings
  - Small sample sizes
  - Limited statistical significance

FISHER'S STATISTIC

$$F = -\sum_{i=1}^{N} 2\log_e(P_i)$$

Single Experiment P-Value

H_0: P_i is uniformly distributed in the interval [0 - 1.0]

⬇

Implies

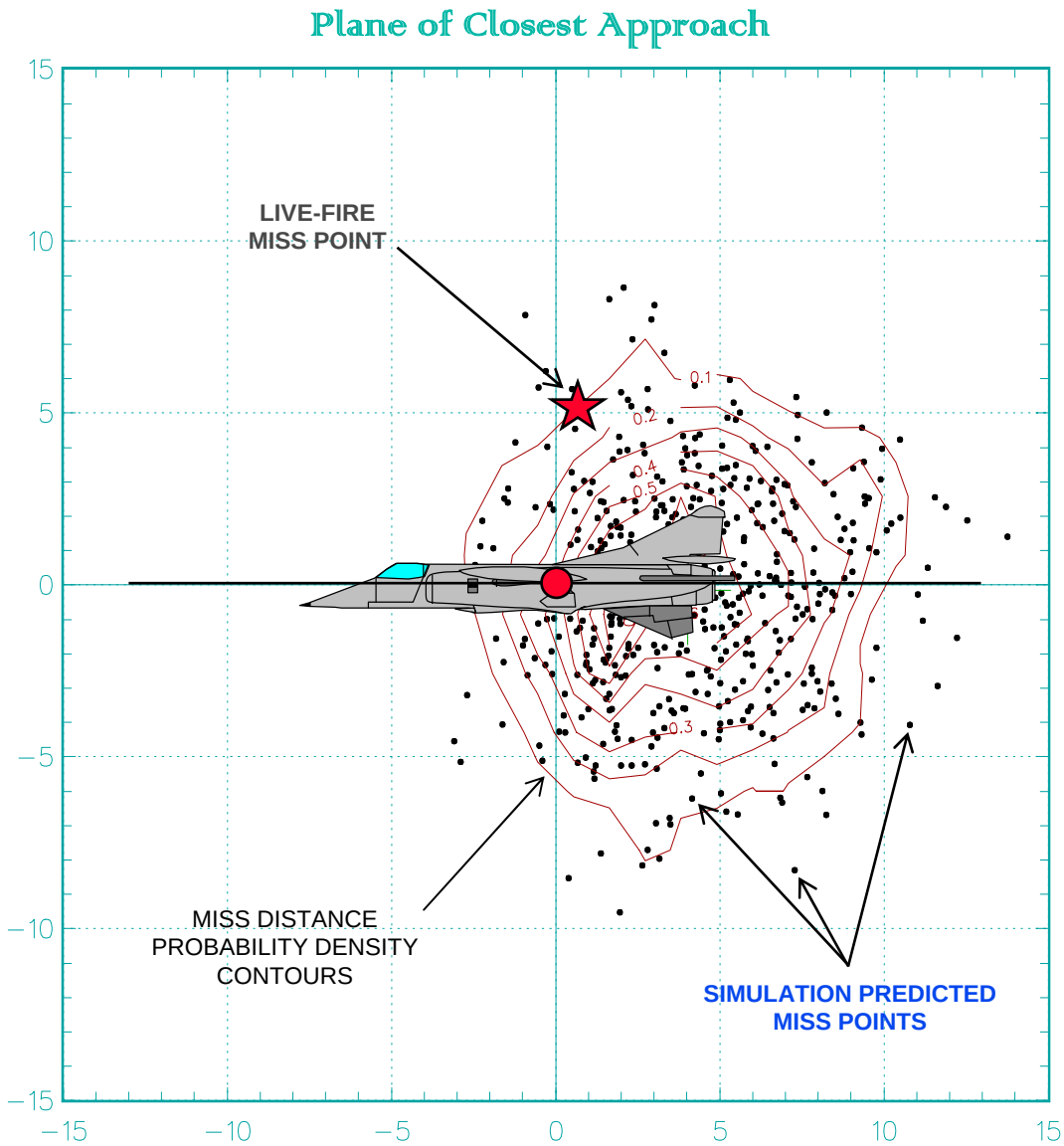
⬇

F is χ^2 distributed with $2N$ Degrees of Freedom

MISS DISTANCE MEASURE OF AGREEMENT

Definition: $\mathbf{P_i}$ (Tail Probability). Fraction of prediction points outside the tangent point contour.

$$P_i = 0.16$$



FISHER'S STATISTIC

$$F = -\sum_{i=1}^{N} 2\log_e(P_i)$$

Single Tail Probability

H_0: P_i is uniformly distributed in the interval [0 - 1.0]

⬇

Implies

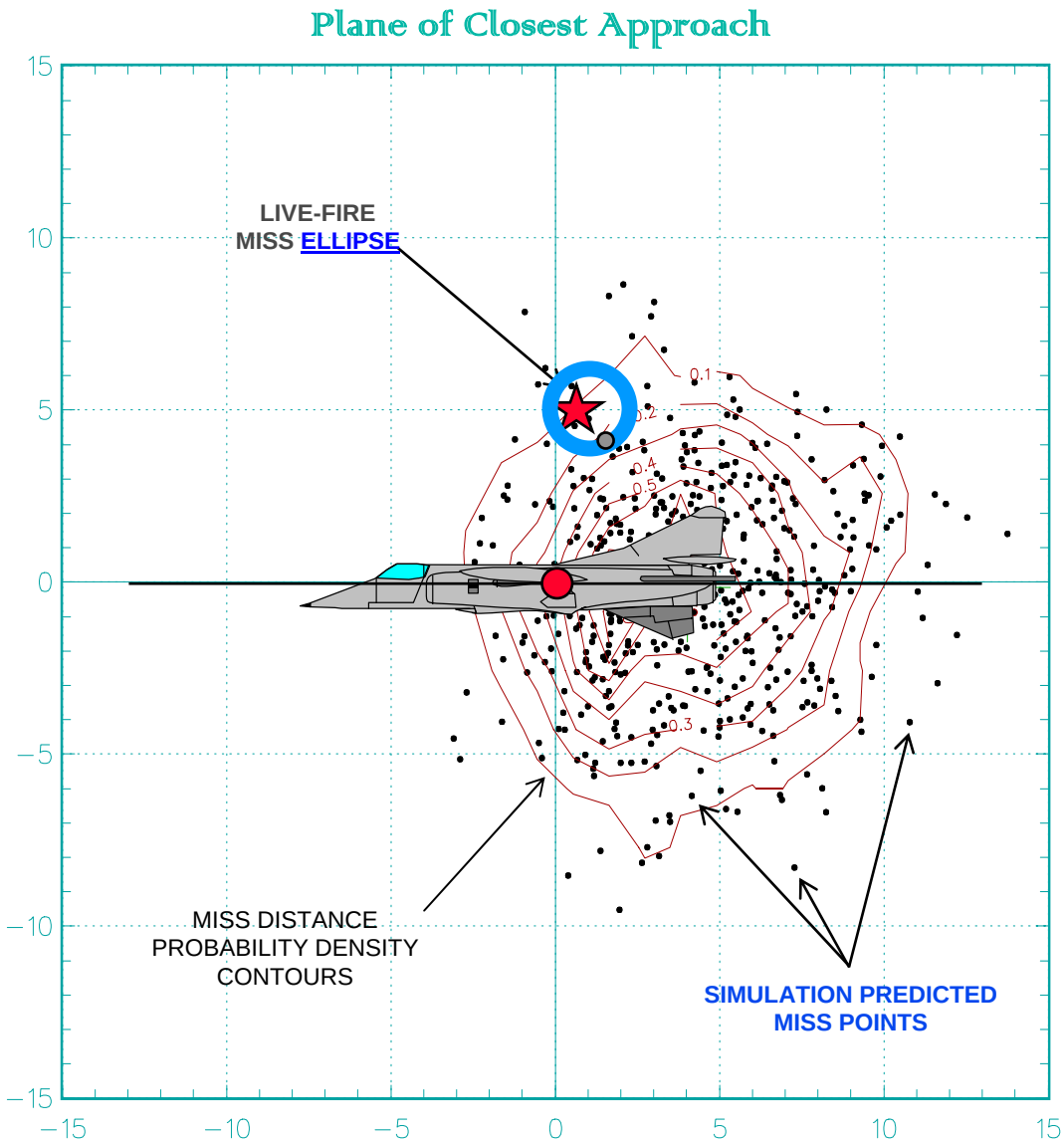
⬇

F is χ^2 distributed with $2N$ Degrees of Freedom

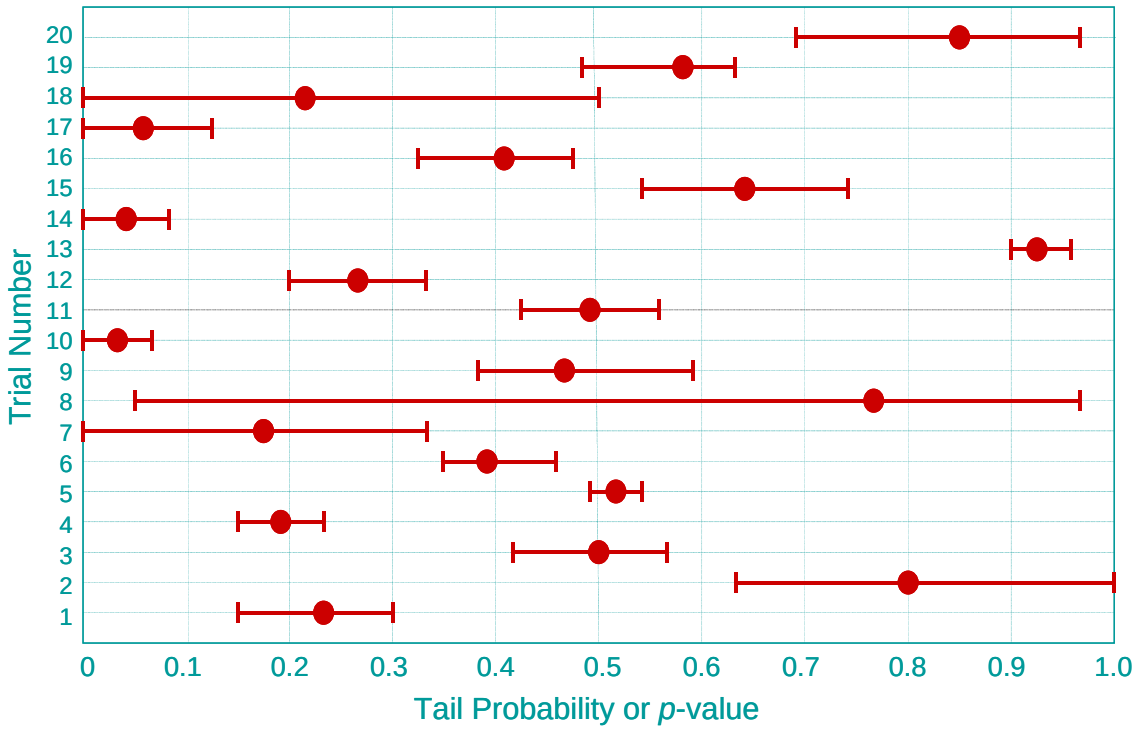
MISS DISTANCE MEASUREMENT ERRORS

Definition: $\mathbf{P_i}$ (Tail Probability). Fraction of prediction points outside the tangent point contour.

$P_i = 0.02$ - 0.28

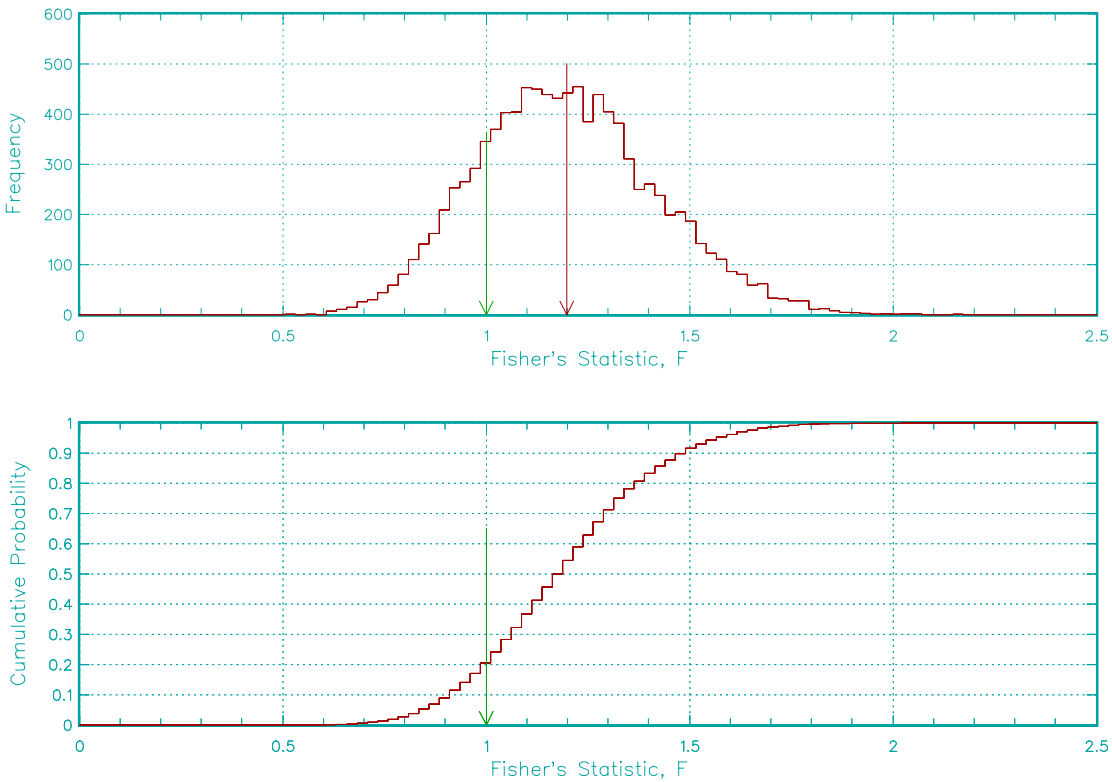


ENCOMPASSING MEASUREMENT ERRORS



Derived Tail Probabilities (*p*-values)

BOOTSTRAPPED FISHER STATISTICS



Distribution of 10,000 *F*-Values and Cumulative Probability

WHICH TAIL(S)?

- **Far-out ↔ Optimistic model**

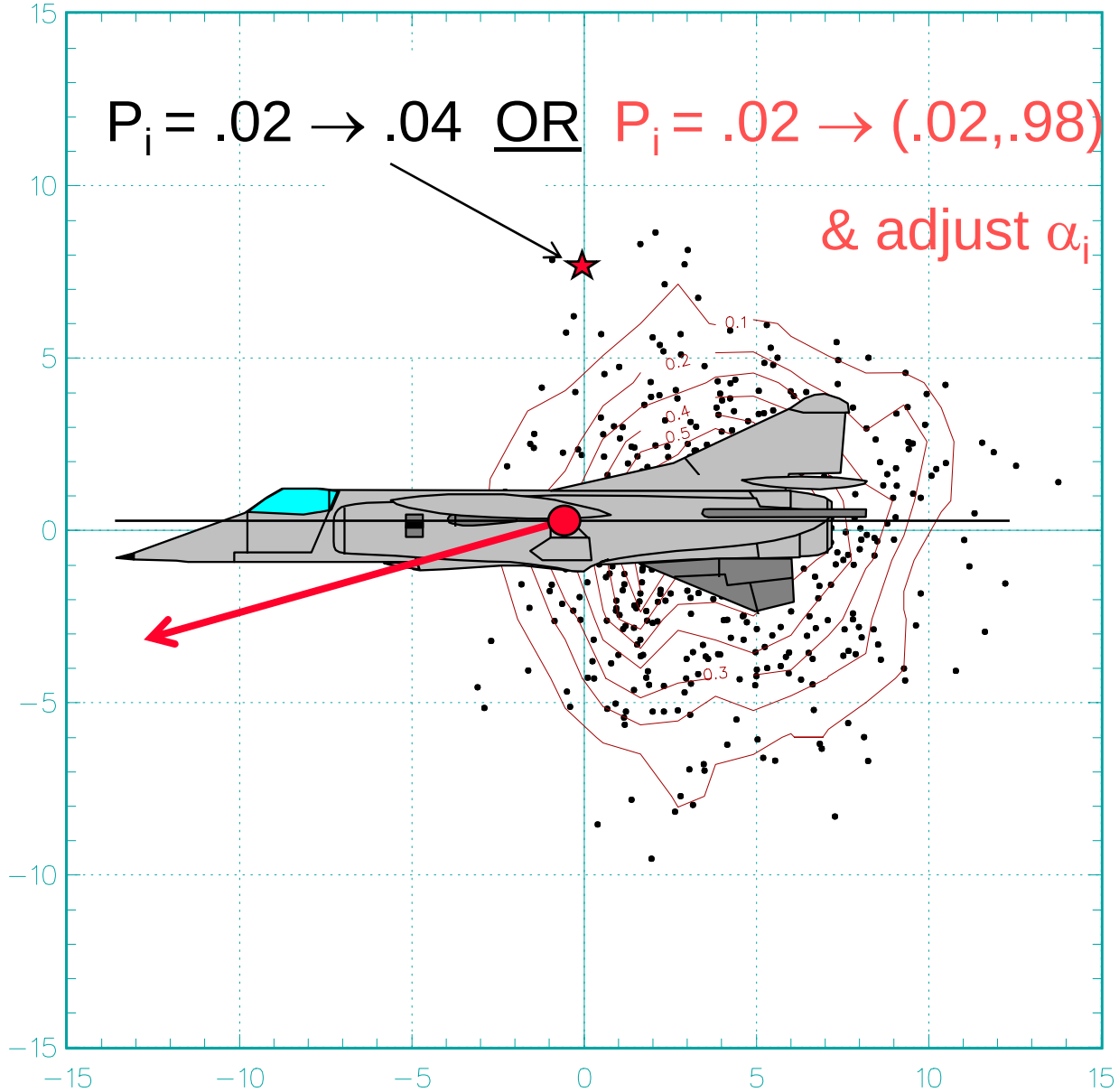
  (big misses are "bad")

- **Close-in ↔ Pessimistic model**

  (small misses are "bad")

- **Both tails ↔ Both concerns**

CONVERSIONS TO 2-SIDED P_i



EXAMPLE CALCULATIONS

- **Simulation Model Optimistic?**
  - $X = 2$ [- ln(.12) - ln(.37) - … - ln(.39)] = 14.4
  - X_O << $\chi^2_{16}(0.05) = 26.3 \Rightarrow$ NO!
- **Simulation Model Pessimistic?**
  - $X = 2$ [- ln(.88) - ln(.63) - … - ln(.61)] = 15.2
  - X_P << $\chi^2_{16}(0.05) = 26.3 \Rightarrow$ NO!
- **Simulation Model "Bad"?**
  - $X = \max(X_O, X_P) << \chi^2_{16}(\underline{0.025}) = 28.8 \Rightarrow$ NO!

META-ANALYSIS APPROACHES

- **<u>One</u> "Outlier" Test Result & Fisher ⇒ "Invalid"**
- **Other <u>Tempered</u> Meta-Analysis Procedures**
- **Endorse Classical Fisher Methodology**
  - Encourages investigation of "outliers"
    - » Test anomaly?
    - » Physical causes represented in model?
  - Promotes discussion
    - » Concept of "invalid"
    - » Statistical significance" vice "practical significance"

GOODNESS-OF-FIT PROCEDURES

- **P_i's Consistent With A Uniform Distribution?**
  - Fisher procedure weights small values heavily
    - » $-\ln(p) \uparrow \infty$ as $p \downarrow 0$
  - Other standard goodness-of-fit tests
    - » Maximum difference in cdf's
    - » Averaged difference

STATISTICAL POWER

- **When the Simulation Model Is "Good"**
  - $S \equiv T$, for all factor combinations
  - With high confidence, should not reject H_0
  - Should attain nominally prescribed α's
- **When the Simulation Model Is "Bad"**
  - $S \neq T$, for some/all factor combinations
  - With high confidence, should reject H_0
  - More "powerful" procedures reject more often

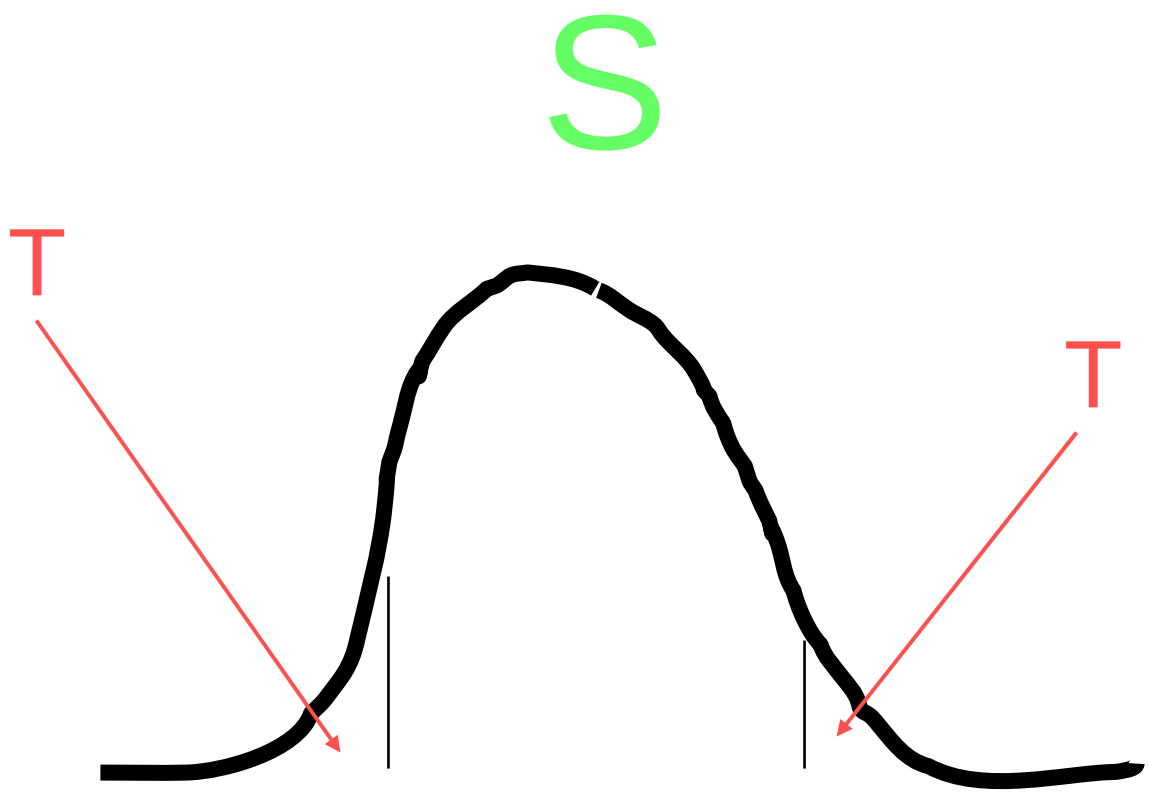
MONTE CARLO ANALYSES

- **Assume "w.l.o.g." $S_i \sim n(0,1)$**
  - Each S_i is "known"
- **Consider $T_i \sim [n(\mu_i,\sigma_i^2)]^{**}\rho_i$**
  - Change mean, variance, shape
  - *t*-test (*Z*-test) optimal for $\mu_i = \mu$, $\sigma_i = \rho_i = 1$
- **Statistical Characterizations**
  - *t*-test (2-sample paired)
  - Fisher
  - Goodness-of-fit

FALSE REJECTIONS

- **By construct, all procedures attain pre-set α's**
- ***t*-test relies on "known" $S_i \sim n(0,1)$**
  - Could generalize to any $n(\mu_i,\sigma_i^2)$
  - But not to "irregular" prediction distributions
- **Other procedures rely merely on "known" S_i**
  - Appropriate for any prediction distribution

VALID REJECTIONS - CASE 1

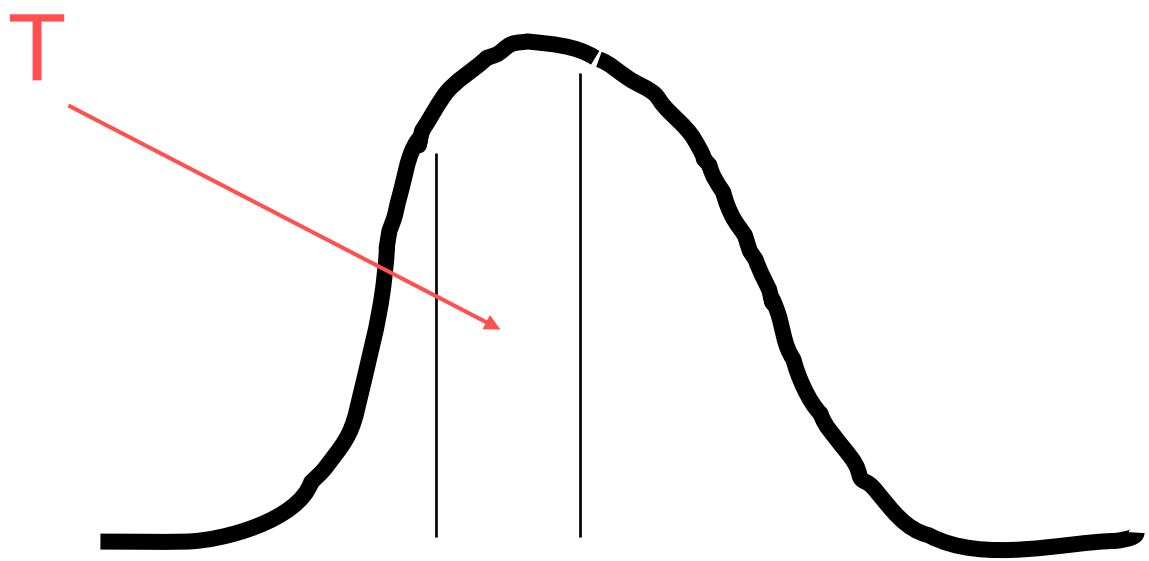


Fisher: Powerful for T in the tails of S

$(\pm \mu, + \sigma^2, + \rho)$

VALID REJECTIONS - CASE 2

S



Fisher: Weak for T in “middle” of S

$(-\sigma^2, -\rho)$

POWER - CASE 1

- **Fisher Dominates**
  - –Fisher >> t-test
  - –Fisher > ≈ GOF's

- **Exception: Shift μ only**
  - –t-test is "optimal"
  - –Fisher is relatively efficient
    - » $S \sim \mathbf{n}(0,1)$, $T \sim \mathbf{n}(0.5, 1)$, N = 25
      - *t*-test 0.88 0.78 0.67 0.40
      - Fisher 0.85 0.77 0.65 0.39

POWER - CASE 2

- ***S* ~ n(0,1), N = 25, α = 0.05**
  - » *T* ~ **n**(0, 0.6^2), Fisher = 0
    - GOF 0.55-0.83
    - t-test 0.05
  - » *T* ~ **n**(0.4, 0.6^2), Fisher = 0.04
    - GOF 0.21-0.38
    - t-test 0.41

- **GOF >> Fisher**

- **GOF > ≈ t-test**

“VALIDATION” CONCERNS

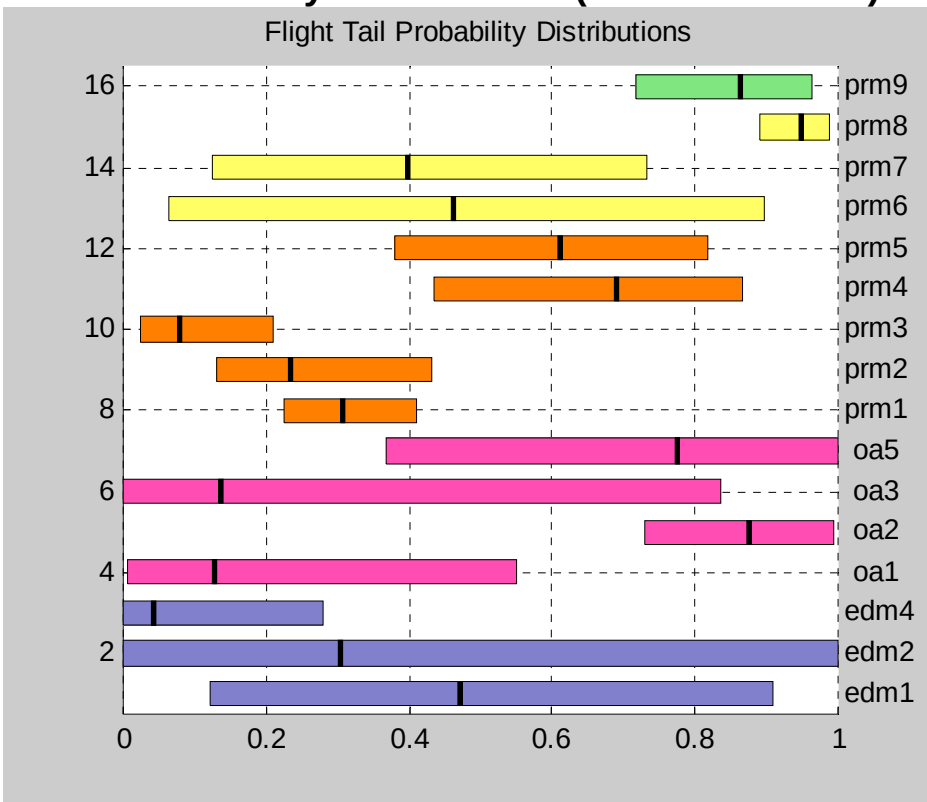
- **Case 1 >> Case 2**
  - –Optimistic Model!!
  - –Pessimistic Model??

LIKELIHOOD OF CASE 2?

- **1-Dimensional Model**
  - –Accurate μ's
  - –Overestimation of σ's
- **Multi-Dimensional Model**
  - –Annuli

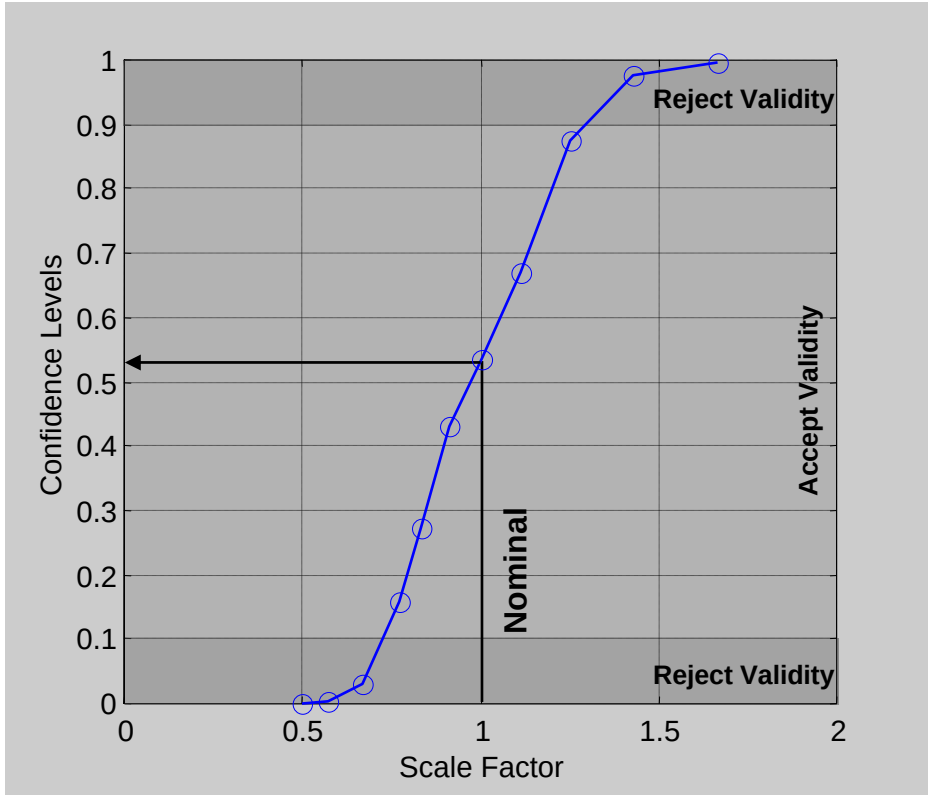
APPLICATION TO AIM-9X

Tail Probability Distributions (w/ uncertainties)



Tail probabilities distributed uniformly from 0 to 1 indicate simulation miss distance is valid

Summary Fisher Miss Distance Results



A Confidence level between 0.1 and 0.9 for nominal miss distance scaling factor (SF = 1) indicates sim miss distance prediction is valid

AIM-9X M&S VALIDATION HISTORY

- **Don't Need To Do It**
- **Can't Do It**
- **Can Only Test At One Design Point**
- **Will Do It!**
  - Implement rigorous managerial procedures & controls
  - Continually inform all T&E organizations
  - Utilize Fisher methodology
- **Awards From DoD Modeling & Simulation Office**
  - Government M&S award for 2001
  - Contractor M&S award for 2001
- **Recent Flight Test Anomaly**

OTHER CHALLENGES

- **Non-Point Predictions**
  - Curves Generated
  - No Single Primary Point Of Emphasis
- **Deterministic Models**
  - Single Prediction For Any Set Of Inputs