SYSTEM TEST TIME BASED ON LINDSTROM/MADDEN APPROACH FOR CONTINUOUS DATA WITH WEIGHTED SUBSYSTEMS

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OBJECTIVES

- DESCRIBE THE METHOD
- SHOW AN EXAMPLE
- EXPLAIN SOME EXCEL ROUTINES
- SHOW HOW THIS METHOD WAS USED TO DETERMINE THE SAMPLE SIZE

DESCRIPTION

Method for computing the Lower Confidence Bound (LCB) on system reliability for independent discrete subsystems in series

LINDSTROM-METHOD FOR COMPUTING LCB'S

- **Step 1**: For each subsystem, calculate the Maximum Likelihood Estimate (MLE) of reliability, $\hat{\mathbf{R}}$, by dividing the number of successes by the number of trials, **N**
- **Step 2**: Calculate the MLE of the system reliability by taking the product of all the MLEs of subsystem reliabilities
- **Step 3**: Obtain the equivalent number of trials **(N)** for the system by choosing the minimum "number of trials" for each individual subsystem
- **Step 4**: Calculate the equivalent number of successes and failures for the system by multiplying **N** by $\hat{\mathbf{R}}$ and **N** by $\mathbf{(1 - \hat{R})}$
- **Step 5**: Obtain the $(1 - \alpha) * 100\%$ LCB for the series system as the $(1 - \alpha) * 100\%$ binomial LCB for a single component with F failures in N tests

"EXAMPLES"
LINDSTROM/MADDEN METHOD
(DISCRETE DATA)

OBJECTIVE IS TO DETERMINE THE SYSTEM RELIABILITY 90% LCB WHEN THERE ARE TWO OR MORE SUBSYSTEMS IN SERIES

EXAMPLE 1	EXAMPLE 2
$R1_{HAT} = 20 / 20 = 1.00$ $R2_{HAT} = 45 / 50 = 0.90$	$R1_{HAT} = 9 / 10 = 0.90$ $R2_{HAT} = 40 / 50 = 0.80$ $R3_{HAT} = 5 / 5 = 1.00$
$R_{SYSTEM} = 1.00 * 0.90 = 0.90$	$R_{SYSTEM} = 0.90 * 0.80 * 1.00 = 0.72$
SYSTEM N MIN (20, 50) = 20	SYSTEM N MIN (10, 50, 5) = 5
SYSTEM S = 20 *0.90 = 18.0	SYSTEM S = 5 * 0.72 = 3.6
90% LCB (18, 20) FOR THE SYSTEM = **0.75523**	90% LCB (3.6, 5) FOR THE SYSTEM = **0.34394**

Reference: Lloyd, D. K., and Lipow, M., "Reliability: Management, Methods, and Mathematics", Prentice Hall, NJ, 1962, p. 227.

BACKGROUND INFORMATION

EXCEL ROUTINE FOR DETERMINING BINOMIAL LOWER AND UPPER CONFIDENCE BOUNDS

EXAMPLES:

EXAMPLES	NUMBER OF SUCCESSES	NUMBER OF TRIALS	90% LOWER CONF BOUND	90% UPPER CONF BOUND
#1	18	20	0.75523	0.97309
#2	3.6	5	0.34394	0.94921

LOWER CONFIDENCE BOUND

= betainv (1 – conf level, # of successes, # of failures + 1)

= betainv (.1, 18, 3) = **0.75523 (Example #1)**

= betainv (.1, 3.6, 2.4) = **0.34394 (Example #2)**

UPPER CONFIDENCE BOUND

= betainv (conf level, # of successes +1, # of failures)

= betainv (.9, 19, 2) = **0.97309 (Example #1)**

= betainv (.9, 4.6, 1.4) = **0.94921 (Example #2)**

"EXAMPLE"
LINDSTROM/MADDEN APPROACH
(CONTINUOUS DATA)

OBJECTIVE IS TO DETERMINE THE SYSTEM RELIABILITY 80% LCB WHEN THERE ARE TWO OR MORE SUBSYSTEMS IN SERIES

Lindstrom-Madden Method					
(Fixed Configurations)					
		α =0.2	yellow = input		
		(1 - α) LCL = **8.09**			
		Sys Point Estimate =**10.00**			
		Number of Subsystems =	4		
n sys =	λ sys * min =	20.00			
λ sys =	Σ w λ =	0.1		Min (Test Times / weights) =	200
Utilization		Weight			Test Time
Factor		times			divided by
(Weight)	λ hat	λ hat	Failures	Test Time	Weight
1.0000	0.05000	0.05000	20	400	400
0.5000	0.04000	0.02000	4	100	200
0.5000	0.02000	0.01000	4	200	400
0.2000	0.10000	0.02000	5	50	250

SYSTEM POINT ESTIMATE = 200 / 20 = 10.00

80% LCB = (2 * 200) / CHIINV (.2, 42) = 400 / 49.45596 = 8.09

Link

ORIGINAL TEST REQUIREMENT

DEMONSTRATE SYSTEM REQUIREMENT OF 66 HOURS AND SUBSYSTEM REQUIREMENTS WITH 80% CONFIDENCE BASED ON THE FOLLOWING ASSUMED USAGE RATES

SUBSYSTEM	ASSUMED USAGE RATE	MTBF REQUIREMENT (HOURS)
A	1.00 (39/39)	170
B	.31 (12/39)	100
C	.15 (6/39)	60
D	.18 (7/39)	50

METHOD USED TO DETERMINE THE NUMBER OF SYSTEM TEST HOURS BASED ON THE ORIGINAL TEST REQUIREMENT

Requirement	Weight	Allowable # of Failures	Subsystem Test Time Required to Demonstrate 80% LCB	System Test Time Required to Demonstrate Subsystem 80% LCB
170	1.0000	2	727	727
100	**0.3077**	**2**	**428**	**1391**
60	0.1538	2	257	**1669**
50	0.1795	2	214	1192

The minimum system test time required to demonstrate each subsystem requirement with at least 80% confidence is ***1669*** hours

Requirement = (2 * Subsystem Test Time) / (CHIINV (.2, 6)
Subsystem Test Time = (CHIINV (.2, 6) * Requirement) / 2
= (8.558058 * 100) / 2
= 428

USING A TEST TIME OF 1669 HOURS, THE MTBF REQUIREMENTS FOR THE SYSTEM AND SUBSYSTEMS WILL BE MET WITH AT LEAST 80% CONFIDENCE WHEN THE NUMBER OF FAILURES IS LESS THAN OR EQUAL TO 2

Lindstrom-Madden Method						
(Fixed Configurations)						
	α =	0.2				
	(1 - α) LCL =	**146.66**				
	Sys Point Estimate =	**208.63**				
	Number of Subsystems =			4		
n sys =	**λ sys * min =**	8.00				
λ sys =	**Σ wλ =**	**0.0048**	**Min (Test Times / weights) =**		**1669**	
Utilization		**Weight**			**Test Time**	
Factor		**times**			**divided by**	
(Weight)	**λ hat**	**λ hat**	**Failures**	**Test Time**	**Weight**	**Requirement**
1.0000	0.00120	0.00120	2	1669	1669	170
0.3077	0.00389	0.00120	2	514	1669	100
0.1538	0.00779	0.00120	2	257	1669	60
0.1795	0.00668	0.00120	2	300	1669	50

IF THE SUBSYTEM TEST TIMES ARE IN ACCORDANCE WITH THEIR UTILIZATION FACTORS (WEIGHTS), THE LINDSTROM MADDEN METHOD IS NOT NECESSARY FOR DETERMINING THE LCB.

REVISION TO THE ORIGINAL TEST REQUIREMENT FOR THE SUBSYSTEMS

- **DEMONSTRATE EACH SUBSYSTEM REQUIREMENT AS A POINT ESTIMATE WHEN THE TEST TIME IS AT LEAST TWICE ITS REQUIREMENT (I.E., THERE ARE NO MORE THAN TWO (2) FAILURES FOR EACH SUBSYSTEM)**

AND

- **DEMONSTRATE THE SYSTEM REQUIREMENT OF 66 HOURS WITH AT LEAST 80% CONFIDENCE.**

METHOD USED TO DETERMINE THE NUMBER OF SYSTEM TEST HOURS BASED ON THE REVISION TO THE ORIGINAL TEST REQUIREMENT

Subsystem	Requirement	Weight	Allowable Number of Failures	Subsytem Test Time Required to Demonstrate the Point Estimate	System Test Time Required to Demonstrate the Point Estimate
A	170	1.0000	2	340	340
B	100	0.3077	2	200	650
C	60	0.1538	2	120	780
D	50	0.1795	2	100	557

The minimum system test time required to demonstrate each subsystem's requirement as a point estimate would be equal to ***780*** hours.

USING A SYSTEM TEST TIME OF 780 HOURS AND NO MORE THAN TWO FAILURES FOR EACH SUBSYSTEM, THE MTBF POINT ESTIMATE FOR EACH SUBSYSTEM IS MET AND THE SYSTEM MTBF REQUIREMENT (66 HOURS) IS MET WITH AT LEAST 80% CONFIDENCE.

Linstrom-Madden Method						
(Fixed Configurations)						
	α =	0.2				
	(1 - α) LCL =	**68.54**				
	Sys Point Estimate =	**97.50**				
	Number of Subsystems =			4		
n sys =	**λ sys * min =**	8.00				
λ sys =	**Σ wλ =**	**0.0103**	**Min (Test Times / weights) =**		**780**	
Utilization		**Weight**			**Test Time**	
Factor		**times**			**divided by**	
(Weight)	**λ hat**	**λ hat**	**Failures**	**Test Time**	**Weight**	**Requirement**
1.0000	0.00256	0.00256	2	780	780	170
0.3077	0.00833	0.00256	2	240	780	100
0.1538	0.01667	0.00256	2	120	780	60
0.1795	0.01429	0.00256	2	140	780	50

CONCLUSIONS

- **TOTAL SYSTEM TEST TIME REQUIRED IS 780 HOURS**

- **WITH NO MORE THAN 2 FAILURES FOR EACH SUBSYSTEM, THE 80% LCB FOR THE SYSTEM MTBF WILL BE GREATER THAN ITS REQUIREMENT (66 HOURS) AND THE POINT ESTIMATE FOR EACH SUBSYSTEM WILL MEET ITS REQUIREMENT.**

- **THE LINDSTROM/MADDEN APPROACH CAN BE USED TO DETERMINE SYSTEM TEST TIME WHEN REQUIRED TO DEMONSTRATE THE SYSTEM REQUIREMENT WITH CONFIDENCE AND THE SUBSYSTEM REQUIREMENTS WITH CONFIDENCE OR AS A POINT ESTIMATE.**

BofM

System Level Analysis vs Subsystem Level Analysis When Testing According to OMS/MP

No. Subsystems = k
Total System Test Time = T
Subsystem Utilization Factor (Weight) = w_i
Subsystem Test Time = $w_i T = T_i$
Subsystem # failures = f_i
System # failures = $\sum_{i=1}^{k} f_i = f$

System Level Analysis: $\hat{MTBF}_{SYS} = \frac{T}{f}$

$\gamma\%$ LCB on MTBF = $\frac{2T}{\chi^2_{\gamma, 2f+2}}$

Subsystem Level Analysis

Subsystem failure rate MLE = $\hat{\lambda}_i = \frac{f_i}{T_i}$

System failure rate MLE = $\hat{\lambda}_{SYS} = \sum w_i \hat{\lambda}_i$

$\hat{MTBF}_{SYS} = \frac{1}{\hat{\lambda}_{SYS}} = \frac{1}{\sum_{i=1}^{k} w_i \lambda_i} = \frac{1}{\sum_{i=1}^{k} w_i \left(\frac{f_i}{T_i}\right)} = \frac{1}{\sum w_i \left(\frac{f_i}{w_i T}\right)} = \frac{T}{f}$ (Same as System Level Analysis)

$\gamma\%$ LCB on MTBF (LINDSTROM/MADDEN APPROACH)

Equivalent System Time = $\min\left(\frac{T_i}{w_i}\right) = \min\left(\frac{w_i T}{w_i}\right) = T$

Equivalent # System Failures = $\hat{\lambda}_{SYS} T = \sum_{i=1}^{k} w_i \hat{\lambda}_i T = \sum_{i=1}^{k} \frac{T_i}{T} \frac{f_i}{T_i} T = \sum_{i=1}^{k} f_i = f$

$\therefore \hat{MTBF}_{SYS}$ $\gamma\%$ LCB = $\frac{2T}{\chi^2_{\gamma, 2f+2}}$ (Same as System Level Analysis)