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Office of Ordnance Research

PROCEEDINGS OF THE FIFTH CONFERENCE ON THE DESIGN OF EXPERIMENTS IN ARMY RESEARCH DEVELOPMENTS AND TESTING

OFFICE OF ORDNANCE RESEARCH, U. S. ARMY
BOX CM, DUKE STATION
DURHAM, NORTH CAROLINA

OFFICE OF ORDNANCE RESEARCH
Report No. 60-2
October 1960

PROCEEDINGS OF THE FIFTH CONFERENCE ON THE DESIGN OF EXPERIMENTS IN ARMY RESEARCH DEVELOPMENT AND TESTING

Sponsored by the Army Mathematics Steering Committee
conducted at
The U. S. Army Biological Warfare Laboratories
Fort Detrick, Frederick, Maryland
4-6 November 1959

OFFICE OF ORDNANCE RESEARCH, U. S. ARMY
BOX CM, DUKE STATION
DURHAM, NORTH CAROLINA

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* This paper was presented at the conference. It is not published in these Proceedings.

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* This paper was presented by title. It does not appear in this technical manual.

** This paper was presented by title.

FOREWORD

The present series of Conferences on the Design of Experiments are sponsored by the Army Mathematics Steering Committee (AMSC). The first three annual meetings were held at the Diamond Ordnance Fuze Laboratories and the National Bureau of Standards in Washington, D. C., and the fourth meeting was conducted at the Quartermaster Research and Engineering Center at Natick, Massachusetts. At its April 1959 meeting the AMSC accepted the invitation, issued by Dr. Clifford J. Maloney on behalf of the U. S. Army Biological Warfare Laboratories, to hold the Fifth Conference on the Design of Experiments at Fort Detrick, Maryland.

The purpose of these Conferences is to afford Army scientific and technological experts an opportunity to exchange views and experiences on problems of designing experiments in research, development and testing, and to learn about new developments in the field from experts in the design of experiments. The success of these Conferences has been due, in large measure, to the interaction and cooperation of these two groups of experts.

The Fifth Conference was attended by 169 registrants and participants from 60 organizations outside of the Biological Laboratories. In addition the host had 71 of its personnel present. Speakers and panelists came from Advanced Research Projects Agency, Bureau of Ships of the Department of the Navy, Mayo Clinic, National Bureau of Standards, Princeton University, RCA Missile Test Project, University of California, University of Georgia, University of Michigan, University of Toronto, Virginia Polytechnic Institute and 15 Army facilities.

This volume of the Proceedings contains 27 of the papers which were presented at the conference. In addition, it contains one of the two articles that were presented by title. The papers are being made available in this form as a contribution to wider dissemination and use of modern statistical principles of the design of experiments in research, development, and testing work of concern to the Army.

The members of the Army Mathematics Steering Committee take this opportunity to express their thanks to the many speakers and other research workers who participated in the meeting; to Colonel Clyde Westbrook, Commanding Officer of the U. S. Army Biological Warfare Laboratories, for making available the excellent facilities of his organization for the Conference; and to Dr. Clifford J. Maloney who handled the details of the local arrangements for the meeting, which included interesting tours of the Laboratories and of nearby Civil War battlefields such as Gettysburg, Antietam and Harper's Ferry.

Finally, the Chairman wishes to express his appreciation to his Advisory Committee, F. G. Dressel (Secretary), Frank E. Grubbs, Boyd Harshbarger, Clifford J. Maloney, and W. J. Youden for their help in organizing the program of the Conference.

S. S. WILKS
Professor of Mathematics
Princeton University

FIFTH CONFERENCE ON THE DESIGN OF EXPERIMENTS IN ARMY RESEARCH DEVELOPMENT AND TESTING

Wednesday AM 4 November

0830 - 0900	REGISTRATION: Post Theater
0900 - 1145	GENERAL SESSION I: Post Theater
	Chairman Dr. I. R. Hershner, Jr., Army Research Office; Office, Chief of Research and Development.
0900 - 0910	Welcome Col. Donald G. Grothaus, Commanding Officer, Fort Detrick.
0910 - 0925	Introductory Remarks Dr. Leroy D. Fothergill, Scientific Advisor, Fort Detrick.
0925 - 0930	Announcements Dr. Morton Reitman, Technical Information Div., Fort Detrick.
0930 - 1030	The Method of Paired Comparisons Dr. H. A. David, Virginia Polytechnic Institute.
1030 - 1045	Break
1045 - 1145	The Measure of Death Dr. Joseph Berkson, Mayo Clinic.
1200 - 1300	LUNCH: Officers' Club

Wednesday PM 4 November

1300 - 1700	TOUR: Battlefield tour of Gettysburg or Antietam and Harpers Ferry. (Buses will depart from the Officers' Club) Harpers Ferry National Monument: Superintendent, Mr. Frank H. Anderson Historian, Mr. Charles Snell Antietam National Battlefield Site: Superintendent, Mr. H. W. Doust Historian, Mr. R. L. Lagemann Gettysburg National Military Park: Superintendent, Mr. James Myers Historian, Mr. Frederick Tilberg

Wednesday PM (Cont'd)

1800 - 1900 SOCIAL HOUR: Officers' Club

1900 - 2000 DINNER: Officers' Club

2000 - 2200 GENERAL SESSION II: Officers' Club

Chairman: Dr. Clifford J. Maloney, Chief, Mathematics Division, Fort Detrick.

2000 - 2100 The Army Research and Development Program as it Relates to the Civil Economy
Dr. Richard Weiss, Army Research Office, Arlington Hall Station, Va.

2100 - 2200 Prediction of the Reliability of Complex Systems
Dr. Nicholas E. Golovin, Director, Technical Operations Division, Advanced Research Projects Agency.

There will be one Clinical and three Technical Sessions conducted Thursday morning. Technical Session I and Clinical Session A will both be held from 0830 - 1040. From 1100 - 1230 Technical Sessions II and III will be running concurrently. The security classification of the first paper in Technical Session III is CONFIDENTIAL. No clearances are required for any of the other papers on this program.

Thursday AM
5 November

0830 - 1040 TECHNICAL SESSION I: Post Theater

Chairman: Mr. Elwood K. Wolfe, Technical Evaluation Division, Fort Detrick.

0830 - 0910 On the Repeated-Measurements Design in Biological Experiments
Ardie Lubin, Department of Clinical and Social Psychology, Division of Neuropsychiatry, Walter Reed Institute of Research, WRAMC.

0910 - 0950 Design of Experiments Using Germfree Animals
Stanley M. Levenson, Ole J. Malm, and Captain Richard E. Horowitz, Department of Surgical Metabolism and Physiology, and the Department of Germfree Research, Walter Reed Army Institute of Research, WRAMC.

0950 - 1005 Break

TECHNICAL SESSION I: (Cont'd)

1005 - 1040	The Development of Parameters for Determining the Resistance of Selected Missile Components to Microbiological Deterioration C. Bruce Lee, Physical Sciences Laboratory, Research and Engineering Directorate, Ordnance Tank-Automotive Command.
1040 - 1100	Break
0830 - 1040	CLINICAL SESSION A: Class Room, Bldg. T-833
	Chairman: Mr. O. P. Bruno, Surveillance Branch, Weapon Systems Laboratory, Ballistic Research Laboratories.
	Panel Members: Besse Day, Bureau of Ships, Dept. of the Navy Frank Grubbs, Weapon Systems Laboratory, Ballistic Research Laboratories Boyd Harshbarger, Virginia Polytechnic Institute G. M. Jenkins, Princeton University R. G. D. Steel, Mathematics Research Center S. S. Wilks, Princeton University
0830 - 0905	Design of Environmental Experiments for Reliability Prediction A. Bulfinch, Nuclear and Advanced Systems Laboratory, Feltman Research and Engineering Laboratory, Picatinny Arsenal.
0905 - 0940	Multidimensional Staircase Designs for Reliability Studies David R. Howes, U. S. Army Chemical Corps Engineering Command
0940 - 0955	Break
0955 - 1040	Approach to Development Policies Concerning Scheduled and Unscheduled Maintenance: Walton M. Hancock and Randall Cline, The University of Michigan, Willow Run Laboratories, Operations Research Department.
1040 - 1100	Break
1100 - 1230	TECHNICAL SESSION II: Post Theater
	Chairman: Dr. Robert M. Thrall, The University of Michigan

TECHNICAL SESSION II: (Cont'd)

1100 - 1140	Statistical Analysis of Various Parameters of Burning Characteristics of Flare Systems Bossie Jackson, Pyrotechnics Laboratory, Feltman Research and Engineering Laboratory, Picatinny Arsenal.
1140 - 1150	Break
1150 - 1230	A Statistical Evaluation of the Pyrotechnic Electrostatic Sensitivity Tester Everett D. Crane, Pyrotechnic Laboratory, Feltman Research and Engineering Laboratory, Picatinny Arsenal.
1100 - 1230	TECHNICAL SESSION III: Conference Room, Bldg. P-560 Chairman: Mr. B. A. Howard, Jr., Headquarters Ordnance Weapons Command
1100 - 1145	Dispersion Strengthening Analysis of Cermets John M. Woulbroun, Sintered Metals and Ceramics Branch, Rodman Laboratory, Watertown Arsenal.
1145 - 1155	Break
1155 - 1230	Experimental Determination of "Best" Component Levels in Thermal Power Supplies (U). (Contents of talk CONFIDENTIAL) Sheldon G. Levin, Diamond Ordnance Fuze Laboratories.
1230 - 1330	LUNCH: Picnic Lunch, Flair Armory. Buses to the armory will leave from the Officers' Club immediately following Technical Session III. There will be movies following lunch. Afterwards, buses will take you to the departure point for the walking tour.

Thursday PM
5 November

1330 - 1700	TOUR: Walking tour of Frederick, Maryland
1800 - 1900	DINNER: Peter Pan Restaurant. Buses to the restaurant will leave from the Francis Scott Key Hotel at 1730.
1900 - 2115	GENERAL SESSION III: Peter Pan Chairman: Dr. S. S. Wilks, Princeton University
1900 - 2000	Medical Health Statistics Dr. Wilford J. Dixon, University of California Medical Center.

GENERAL SESSION III: (Cont'd)

2000 - 2015	Break
2015 - 2115	Sampling in Biological Populations Dr. D. B. DeLury, University of Toronto.

Friday AM
6 November

0830 - 1040	TECHNICAL SESSION IV: Post Theater
	Chairman: Mr. John P. Purtell, Research Branch, Watervliet Arsenal.
0830 - 0910	The Application of Fractional Factorials in Missile Test Programs Paul C. Cox, Reliability and Statistics Office, Ordnance Mission, White Sands Missile Range.
0910 - 0940	The Design and Re-design of an Experiment C. W. Mullis, Plans Branch, Integrated Range Mission, White Sands Missile Range.
0940 - 0950	Break
0950 - 1015	On a Problem of Misclassification: A. C. Cohen, Jr., The University of Georgia
1015 - 1040	Detecting and Quantifying Guess Responses in the Rating of Statements by a Method of Successive Intervals Lee E. Paul, Methods and Systems Engineering Branch, Quartermaster R and E Field Evaluation Agency.
0830 - 1040	CLINICAL SESSION B: Class Room, Bldg. T-833.
	Chairman: Mr. John Kosar, Missile Warheads and Special Projects Laboratory, FREL, Picatinny Arsenal.
	Panel Members: H. A. Daivd, Virginia Polytechnic Institute D. B. DeLury, University of Toronto W. J. Dixon, University of Cal. Medical Center W. D. Foster, Fort Detrick J. S. Hunter, Mathematics Research Center, U. S. Army W. J. Youden, National Bureau of Standards

CLINICAL SESSION B: (Cont'd)

0830 - 0900	Design for Estimation by Covariance Techniques: Morris Rhian, Aerobiology Division, U. S. Army Biological Warfare Laboratories.
0900 - 0920	Design of an Experiment to Evaluate a Bio-assay with Non-parallel Slopes Albert L. Fernelius, Process Research Division, U. S. Army Biological Warfare Laboratories.
0920 - 0930	Break
0930 - 1010	The ORO Aircraft Vulnerability Experiment Bruce Taylor, Operations Research Office, The Johns Hopkins University
1010 - 1040	Operational Hit Probabilities of Experimental Anti-tank Weapons J. D. Reed, R. E. Tiller, and J. P. Young, Operations Research Office, The Johns Hopkins University.
1055 - 1230	TECHNICAL SESSION V: Post Theater
	Chairman: Dr. H. Leon Harter, Wright Air Development Center, Wright Patterson Air Force Base.
1055 - 1135	Elimination of Bias Introduced by Transformation of Variables Jerzy Neyman and Elizabeth L. Scott, Statistical Laboratory, University of California, Berkeley.
1135 - 1145	Break
1145 - 1230	Mathematical and Statistical Principles Underlying Chemical Corps Inspection Procedures for Product Verification Henry Ellner and Joseph Mandelson, Materiel Command at the Army Chemical Center.
1055 - 1230	TECHNICAL SESSION VI: Class Room, Bldg. T-833.
	Chairman: Mr. Abraham Golub, Support Weapons Evaluation Branch, Weapon Systems Laboratory, Ballistic Research Laboratories.
1055 - 1140	Measuring a Complex Field Operation K. L. Yudowitch, Operations Research Office, The Johns Hopkins University
1140 - 1150	Break

TECHNICAL SESSION VI: (Cont'd)

1150 - 1230 The Conduct of Military Field Research on a Shoe-String
A. J. Eckles, III, and R. E. Zimmerman, Operations Research Office, The Johns Hopkins University.

1230 - 1330 LUNCH: Optional

Friday PM
6 November

1330 - 1500 TOUR: A conducted tour of the Fort Detrick Laboratories to start from the Officers' Club.

SUPPLEMENTARY PROGRAM

We are sorry that time did not permit the scheduling of the following two papers. These authors, as well as all speakers on this program, are urged to submit manuscripts of their papers so that a complete and interesting technical manual can be published. A copy of these Proceedings will be sent to each attendee of this conference.

Sample Order Statistics of the Circular Normal Distribution
Helen J. Coon, Weapon Systems Laboratory, Ballistic Research Laboratories.

Determination of Systematic Errors in Tracking Radar
Victor B. Kovac, RCA Missile Test Project, Patrick Air Force Base.

THE METHOD OF PAIRED COMPARISONS

H. A. David
Virginia Polytechnic Institute

INTRODUCTION. In a paired-comparison experiment objects or "stimuli" are presented in pairs to a panel of judges who act independently. The basic experimental unit is the comparison of two objects, A and B, by a single judge who, in the simplest situation, must state which one he prefers. One may also allow the judge the third alternative of declaring a tie. A further generalization would be to give the judge a scale of preferences; for example, a seven-point scale reading "strong preference for item A," "preference for A," "slight preference for A," "no preference," "slight preference for B," "preference for B," "strong preference for B." These preferences may be scored by assigning object A the score i ($i = 3, 2, 1, 0, -1, -2, -3$) and B the score $-i$. A slightly different scoring system prevails in a widely publicized form of paired comparison such as we have recently been witnessing in the series between the Dodgers and the White Sox where each game corresponds to one paired comparison and the series to several repetitions.

The comparison of A and B may be made by all the judges. If more than 2 objects are to be compared it is still possible to arrange that every judge makes every possible paired comparison either once or several times. This situation may be called a balanced paired-comparison experiment and corresponds in the language of sport to a Round Robin tournament; the roles of the players in the tournament being analogous to those of the objects in the paired-comparison experiment. If we have t objects and n judges the number of paired comparisons will be $\frac{1}{2}rn\, t\, (t - 1)$, where r is the number of times a particular judge makes a particular paired comparison or in other words, the number of replications of a simple Round Robin tournament.

The method of paired comparisons is used primarily in cases when the objects to be compared can be judged only subjectively; that is to say, when it is impossible or impracticable to make relevant measurements in order to decide which of two objects is preferable. As may be inferred, paired comparisons are widely employed by psychometricians, and the method was indeed first introduced by Thurstone (1927). Most frequent applications have been to taste testing, color comparisons, personnel rating, and generally to all forms of preference testing. Of course, there are other methods of sensory discrimination and it is not proposed to enter into a detailed discussion of the individual merits of these methods, particularly as a number of summary accounts have recently been given [Jones and Bock (1957), Torgerson (1958) and Bliss (1959)]. The method of paired comparisons is sometimes the only practicable experimental procedure as in testing various brands of razors where two razors can be compared on a man's two cheeks. Sometimes it may be possible for a judge to compare several objects at the same time and if this can easily be done it would indeed be preferable for the judge to assign ranks to all these objects. However, when differences between objects are small it is advantageous to make the comparison between two of them as free as possible from any extraneous influences such as may be provided by taking into consideration other objects at the same time. Thus the method of paired comparisons will be used in cases where a fine judgment is needed. Again, in taste testing it is often not possible for a judge to cope with more than two tastes, and the introduction of a third taste may be thoroughly confusing.

When both paired comparisons and ranking are possible procedures in arranging several objects in order of preference, ranking will certainly be the speedier. On the other hand, the method of paired comparisons makes it possible for the judge to contradict himself; for example, he may prefer A over B, B over C, and yet C over A. This situation is certainly not impossible and has been called a circular triad by Kendall. An extreme example is provided by the game of stone, scissors, and paper. It is clear that if one judge is guilty of considerably more circular triads than another, then he is a less consistent judge. We have, therefore, a basis for a method for selecting good judges. The explanation of a circular triad may be that the judge is essentially guessing or it may be that in making the three comparisons he changes the criterion on which he bases his judgment. Putting it in different words, the preference scale may well not be uni-dimensional. A preference may be based on a number of characteristics of the objects and presumably these characteristics are weighted in some way in the judge's mind before he comes to a decision. The weights assigned may well vary from comparison to comparison for an inexperienced judge.

In the remainder of this paper we shall consider a number of points arising in the design and analysis of paired-comparison experiments, with special emphasis on some work recently done at the Virginia Polytechnic Institute.

THE DESIGN OF PAIRED-COMPARISON EXPERIMENTS. In the language of the design of experiments a Round Robin tournament is simply a balanced incomplete block design with judges corresponding to replications and with block size 2. Questions of design become more difficult when it is not feasible for every judge to make all possible comparisons. A very considerable degree of balance can sometimes be retained by what Bose (1956) has termed "linked paired comparison designs." An example of such a design is given in Table 1. Even more balance could be obtained if it is important to eliminate effects due to order of presentation within a pair. Related problems are discussed by Kendall (1955). Simpler but less well balanced methods of partial pairing had previously been developed by McCormick and Bachus (1952) in connection with the rating of a large number of employees.

It is a well-established dogma of experimental design that an experiment should contain a large degree of balance. There are, however, situations when balance is a doubtful asset. If we are interested in discovering the best of a number of treatments it is intuitively more reasonable to proceed sequentially - if this is practicable - in a fashion which will result in more intensive testing of those treatments most successful in the early stages of the experiment. Recalling that a balanced paired-comparison experiment is equivalent to a Round Robin tournament we are led to consider other types of tournaments such as the Knock-out which have as their aim picking the strongest of a group of players.

Consider a tournament of 4 players. A simple (i.e. unreplicated) Round Robin tournament requires 6 games, as do <u>two</u> replications of a Knock-out tournament. As a first step toward a wider comparison one may therefore investigate the effectiveness of these two tournaments in determining the best player. This may be done by assigning values to each π_{ij}, the probability that player i defeats player j, and finding the probability that the strongest player (the player for whom $_{i.}$ is largest) will win the tournament. In calculating this probability we average over all possible draws. The situation is unfortunately complicated by the possible need for play-offs if two or three players end up in the lead. In addition to the probability that the best player will win it is therefore advisable to take into consideration the expected number of games required to determine the winner. Both criteria are evaluated in [3] by enumeration of all possible outcomes of the tournament and determination of their probabilities. In a series of examples studied the Knock-out tournament does in fact emerge as superior on both counts in nearly all cases. Another type of tournament employs double elimination; that is, first round losers are paired off and a player is eliminated only after losing to two opponents. This turns out to be the best of three types of tournament. A variation of the Knock-out tournament, which in any match between 2 players requires not one but the best out of 3 games to determine the winner, has been suggested by Maurice (1958). It is not easily compared with the other tournaments, except on the basis of a cost function, as it tends to require more games in return for a higher probability of determining the best players.

The following is typical of the results obtained. With parameter values

$$\pi_{12} = .70,\ \pi_{13} = .76,\ \pi_{14} = .86,\ \pi_{23} = .75,\ \pi_{24} = .82,\ \pi_{34} = .72$$

the probability that player 1 (the strongest) will win and the corresponding expected number of games is

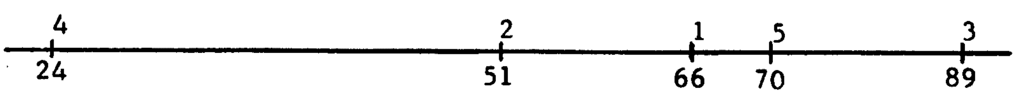
0.644, 6.62	for the Round Robin tournament,
0.656, 6.56	for the Knock-out tournament,
0.686, 6.43	for the Double Elimination tournament,
0.706, 7.08	for Maurice's tournament.

<u>ESTIMATION PROCEDURES</u>. We return now to a more detailed consideration of a balanced paired-comparison experiment in which each of n judges compares t objects r times. Further we suppose that each comparison results in a straight preference for one or the other object judged. The results for each judge can then be fully presented in the familiar

two-way table of 1's and 0's. In addition, the number of times each object is preferred to all others may be listed in a column of totals (the number of wins or score of each object, treatment, or player). If differences between judges can be assumed to be slight - and this can be tested - the n individual tables are conveniently amalgamated into a single summary table. For example, in the pairwise comparison of 5 brands of carbon paper by 30 secretaries (see Fleckenstein et al, 1958, and [2] for details) the following results, condensed from the original 7-point scale used, were obtained:

Brand	1	2	3	4	5	Total a_i
1	-	20	6	25	15	66
2	10	-	10	20	11	51
3	24	20	-	27	18	89
4	5	10	3	-	6	24
5	15	19	12	24	-	70
						300

Generally, the upshot of an experiment of this type has been the construction of a "response scale" in which the objects are appropriately spaced in increasing order of preference along a straight line. An obvious way of doing this is to use the total scores. Thus the results of the carbon paper experiment can represented as follows:



Here only the <u>relative</u> distances between scores are important.

This simple procedure may be regarded as a method of estimation. Let π_{ij} be the probability that in the comparison of objects i and j, i is preferred to j; and let

$$\pi_{i.} = \sum_{\substack{j=1 \\ j \neq i}}^{t} \pi_{ij} \quad (= \textstyle\sum_j{}' \pi_{ij}, \text{ say}).$$

Also let a_{ij} be the observed number of times that i is preferred to j, so that $a_i = \sum_j' a_{ij}$. Then clearly,

$p_{ij} = a_{ij}/n$ is an estimate of π_{ij}

$p_{i.} = a_i/[n(t-1)]$ is an estimate of $\pi_{i.}$.

It is surprising that this simple distribution-free method of estimation has not been more widely used. What has been usually done instead is to propose specific models giving the $\frac{1}{2}t(t-1)$ parameters π_{ij} in terms of t parameters (or t-1 if the origin of the scale is fixed).

Two cases have received special attention:

(1) $$\pi_{ij} = \int_{-(S_i-S_j)}^{\infty} \frac{1}{\sqrt{(2\pi)}} e^{-\frac{1}{2}x^2} dx$$

(Thurstone, 1927; Mosteller, 1951), where the responses to the t objects are assumed to be equi-correlated normal variates with true means $S_i (i = 1,\ldots,t)$ and common variances.

(2) $$\pi_{ij} = \pi_i/(\pi_i + \pi_j)$$

(Bradley and Terry, 1952), where the π_i are true "ratings" of the objects and satisfy $\pi_i \geq 0$, $\Sigma\pi_i = 1$.

If the models are appropraite they will generally lead to better scales than the simple scale above, which is however much more widely valid. (1) and (2) as well as two other scales have been compared by Jackson and Fleckenstein (1957) who found the four scales quite close in a color preference test.

SIGNIFICANCE TESTS. A question that arises naturally in the interpretation of a response scale, whatever its mode of derivation, is whether any differences between objects indicated by the scale are in fact statistically significant. Several methods of constructing over-all tests are available, that is tests of the null hypothesis H_o' that all treatments are alike (in the responses they evoke). The simplest of these tests is to make use of the fact that

$$D = 4 \sum_{i=1}^{t} (a_i - \bar{a})^2 / (nt)$$

is, on H_o', distributed approximately as χ^2 with t-1 degrees of freedom. This is a special case of a more general test given by

Durbin (1951) and is equivalent to an older method based on counting the number of circular triads (Kendall and Smith, 1940). The goodness of the χ^2 approximation is examined in [2]. For the carbon paper experiment

$$D = 4 \times 2,354/30 \times 5 = 62.77,$$

which is a highly significant value of χ^2 with 4 D.F.

This overall test leaves many questions unanswered, for example:

(1) If, prior to the experiment, one of the t objects in the paired-comparison experiment is of particular interest to the experimenter, how can he use the results to test whether this object is better (or worse) than, or different from, the average of all objects?

(2) If, before the experiment, there is a special interest in whether two specified objects produce different responses, how does one use the results of the full paired-comparison experiment to test for a difference?

(3) How does one test whether the object with the highest (lowest) score in the experiment is significantly better (worse) than the average?

(4) How does one order the t objects in a paired-comparison experiment into significantly different groups?

(5) How does one test whether the difference of two treatment scores which are chosen after the completion of the experiment is significant?

To answer questions (4) and (5) it is possible to adapt the well-known multiple comparison procedures due to Tukey and to Scheffé. This approach will not be treated here but is described in [2]. We now consider questions (1) - (3) in turn.

(1) Test of a pre-assigned object

Because of cost of some other characteristic of object r (O_r), $1 \leq r \leq t$, the experimenter may be particularly interested in knowing whether this object is better than average, that is, if

$$\pi_{r.} = \sum_j{}' \pi_{rj}/(t-1) > \tfrac{1}{2}.$$

On H_o' the score a_r of O_r is a binomial variate with parameters $n(t-1)$, $\frac{1}{2}$. If a_r^o is the observed score of O_r the corresponding significance level is

$$\Pr(a_r \geq a_r^o \mid H_o') = 2^{-n(t-1)} \sum_{k=a_r^o}^{n(t-1)} \binom{n(t-1)}{k}.$$

Except in small experiments a normal approximation can be used to evaluate this probability.

In view of the generality of our model the point arises here and elsewhere that one may in fact be interested in testing not H_o' (the hypothesis that all objects are alike) but the more general null hypothesis

$$H_o: \quad \pi_{i.} = \tfrac{1}{2} \qquad \text{all } i.$$

The two hypotheses are the same for the models of Thurstone and Bradley-Terry, and indeed for any linear model. It can be shown [5] that the above procedure is conservative under H_o; that is, the level of significance under H_o' is greater than under H_o.

(2) Tests of equality of two pre-assigned objects

Consider the case in which interest is expressed before the experiment in testing the difference between O_r and O_s. One therefore wishes to test H_o against one-sided or two-sided alternatives $\pi_{r.} > \pi_{s.}$ or $\pi_{r.} = \pi_{s.}$, respectively. This can be done by finding the distribution of $d = a_r - a_s$ under H_o'. Table 2 giving upper 5 and 1% points of d has been constructed from the exact distribution of d for small experiments and a normal approximation (with continuity correction) otherwise.

Illustration. In the carbon paper experiment brand 2 is more expensive than brand 4. Is it significantly better?

A one-sided test is required, say at the 5% level. We have $d = a_2 - a_4 = 51 - 24 = 27$. Also

$$1.64 \sqrt{nt/2} + 0.5 = 1.64 \sqrt{75} + 0.5 = 14.7$$

giving $d_c = 15$. Since d d_c we may declare brand 2 superior to brand 4.

(3) Test of the highest score

After running a paired-comparison experiment, the experimenter may wish to know whether the object with the highest score (a_{max}, say) is significantly better than average.

Let A_i be the event $a_i \geq m \left[0 \leq m \leq n(t - 1)\right]$. Then by the principle of inclusion and exclusion

$$\Pr(a_{max} \geq m) = \Pr\left(\sum_{i=1}^{t} A_i\right)$$

$$= \sum_{j=1}^{t} (-1)^{j-1} \binom{t}{j} \Pr(A_1 A_2 \ldots A_j).$$

For small experiments it is possible to evaluate this probability exactly and tables are given in [1] for n = 1. In other cases it is often adequate to use the first term in the sum, viz.,

$$t\ \Pr(a_i \geq m) = t\ 2^{-n(t-1)} \sum_{k=m}^{n(t-1)} \binom{n(t-1)}{k}$$

as an approximation to $\Pr(a_{max} \geq m)$; it is, of course, also an upper bound. To test the significance of a_{max} approximately at level α one chooses as the critical value that positive integer m, say m_β, for which

$$t\ \Pr(a_i \geq m_\beta | H_o') = \beta \leq \alpha \leq t\ \Pr(a_i \geq m_\beta - 1 | H_o').$$

If $a_{max} \geq m_\beta$ one concludes that the object with score a_{max} is better than average at the 5% level of significance.

Illustration. In the carbon paper experiment brand 3 obtained the highest score: $a_{max} = 89$. To test whether this is significant at the 5% level we note from tables (e.g. Harvard Univ., 1955) that for sample size $n(t - 1) = 120$ and $p = \frac{1}{2}$

$$5\ \Pr(a_i \geq 74) = 0.033$$

and $5\ \Pr(a_i \geq 73) > 0.05$.

Thus $\beta = 0.033$ and $m_\beta = 74$.

Since $a_{max} > 74$, we conclude that brand 3 is significantly better than average.

THE TREATMENT OF TIES. In our discussion of estimation procedures and significance tests we have assumed that judges are not allowed to declare ties. This certainly simplifies the analysis but is frequently not desirable. Various methods for treating ties are in use: equal division among the tied objects, decision by the toss of a coin, and ignoring ties altogether. The last method has advantages in significance testing but is clearly unsuitable for the estimation of a response scale since it does not distinguish between results such as the following: A preferred 4 times, B once, no ties and A preferred 4 times, B once, 20 ties. The other two approaches may seem very plausible but if A is generally preferred to B it is likely, on the whole, to have had a slight edge on B even in those cases where the judge could reach no decision. The following model is proposed in [4]. Suppose that in the comparison of two objects O_i and O_j by a particular judge a response x_i is evoked by O_i and a response x_j by O_j. If $|x_i - x_j| \leq \tau$ the judge declares a tie, if $x_i - x_j > \tau$ he prefers 0_i, if $x_j - x_i > \tau$ he prefers 0_j. Here the symbol τ denotes a sensory threshold. If $\tau = 0$ we are back in the situation where the probability of a tie is zero.

The model can be superposed on that of Thurstone and Mosteller. Least squares methods can then be used to estimate not only the mean responses S_i ($i = 1, \ldots, t$) but also the parameter τ (and possibly different τ's for different judges, a point which can be tested). Actually in [4] the differences $x_i - x_j$ were taken to follow a cosine law rather than the normal law of Thurstone. It should be noted that no splitting of the ties is actually made, the original observations being used in the analysis. The model has been found to give a satisfactory fit in the carbon paper experiment when the original 7-point scale is condensed into a 3-point scale.

Table 1

A linked paired comparison design for 5 treatments and 6 judges

Judge	Pairs assigned to a judge
a	(3, 5), (2, 4), (1, 3), (1, 4), (2, 5)
b	(2, 3), (3, 4), (1, 4), (1, 5), (2, 5)
c	(2, 3), (3, 5), (1, 2), (4, 5), (1, 4)
d	(3, 5), (1, 2), (3, 4), (2, 4), (1, 5)
e	(1, 2), (3, 4), (4, 5), (1, 3), (2, 5)
f	(2, 3), (4, 5), (2, 4), (1, 3), (1, 5)

$t = 5$ (no. of treatments or objects to be compared)

$n = 6$ (no. of judges)

$b = 10$ (no. of different pairs)

$r = 5$ (no. of pairs compared by each judge)

$k = 3$ (no. of times each pair is judged)

$\lambda = 2$ (no. of pairs compared in common by any two judges)

$\alpha = 2$ (no. of times each object is compared by each judge)

(From R. C. Bose (1956) with a slight change in notation)

Table 2

Critical values of d, the difference in scores of two pre-assigned objects (t = no. of objects, n = no. of replications)

Experiment Size		$\alpha = 0.01$ one-sided test	$\alpha = 0.01$ two-sided test	$\alpha = 0.05$ one-sided test	$\alpha = 0.05$ two-sided test
n	t	d'_c	d_c	d'_c	d_c
1	≤ 4	no significant values		no significant values	
1	5	4	none possible	4	4
1	6	5	5	4	4
1	7	5	5	4	5
1	8	5	6	4	5
1	9	6	6	4	5
1	10	6	7	5	5
1	11	6	7	5	6
1	12	7	7	5	6
1	13	7	7	5	6
1	14	7	8	5	6
1	15	7	8	5	6
1	16	7	8	6	6
2	3	no significant values		4	4
2	4	5	6	4	5
2	5	6	6	5	5
3	3	6	6	4	5
3	4	6	7	5	6
4	3	6	7	5	6
4	4	7	8	6	6
All larger values of n or t		$d'_c =$ smallest integer $\geq 2.33\sqrt{\frac{1}{2}nt} + 0.5$	$d_c =$ smallest integer $\geq 2.56\sqrt{\frac{1}{2}nt} + 0.5$	$d'_c =$ smallest integer $\geq 1.64\sqrt{\frac{1}{2}nt} + 0.5$	$d_c =$ smallest integer $\geq 1.96\sqrt{\frac{1}{2}nt} + 0.5$

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MEASURE OF COMPETING EXPONENTIAL MORTALITY RISKS WITH ESPECIAL REFERENCE TO THE STUDY OF SMOKING AND LUNG CANCER

Joseph Berkson, M.D.
Mayo Clinic, Rochester, Minnesota

I shall consider the model of two competing risks in the sense of Neyman [10]; and to set out the problem, I take first a very simple example.

Two marksmen shoot at a range of targets, under conditions in which if a target is struck, it drops instantly from view so that it cannot be struck again. This provision is made because the striking of a target with a bullet is intended to represent the striking down of a man by death from disease. Let the striking rate of Marksman 1 (who may be taken to represent a specific disease), when he is firing alone, be q_1, and similarly let the rate when Marksman 2 is firing alone be q_2. The probability when one risk operates alone is called the "net" risk or rate, and is represented by lower case q; when it operates together with another risk, the resulting risk is called the "crude" risk or rate and is represented by capital Q.

Suppose N targets are exposed and Marksman 1 shoots first, followed by Marksman 2.

(1) Rate for 1 is $Q_1 = q_1$

(2) Rate for 2 is $Q_2 = (1 - q_1)\, q_2$

(3) Total rate is $Q_1 + Q_2 = q_1 + q_2 - q_1\, q_2$

Suppose, instead, Marksman 2 shoots first, followed by Marksman 1.

(4) Rate for 2 is $Q_2 = q_2$

(5) Rate for 1 is $Q_1 = (1 - q_2)\, q_1$

(6) Total rate is $Q = q_1 + q_2 - q_1\, q_2$

It is seen that the total crude rate, with both marksmen firing, is the same, whichever shoots first, and assuming independence of the net probabilities q_1 and q_2, this will be true in general. Regardless of the order of shooting, or whether the two marksmen shoot together, the total crude rate is given by (3) (6). This result would, of course, usually be derived as the complement of the product of the probabilities $p_1 = 1 - q_1$ and $p_2 = 1 - q_2$, of not being struck; that is as 1 minus the product of the survival rates.

If, from independent trial, we knew q_1, the net rate of Marksman 1, and had observed the result Q, the crude rate when both shot together, we could derive the net rate q_2 of Marksman 2 from (3):

$$q_2 = \frac{Q - q_1}{1 - q_1} \tag{7}$$

But suppose we did not know the net rate of either marksman, q_1 of q_2, but had observed the results of their shooting together, and could identify the number of targets struck by each, from the shape of the bullet hole or otherwise, so that we could determine the individual crude rates Q_1 and Q_2 -- still we could not determine the net rates q_1, q_2, from these data alone. We have seen that, with the same net rates q_1, q_2 operating, although the total crude rate Q is independent of the order of shooting, the individual crude rates Q_1, Q_2 depended on which marksman shot first. This problem of estimating a risk, from observations when another risk is operating with it, called "competing risks," by Neyman [10], arises in different contexts of many statistical problems.

In order to estimate the net q's from the observed crude Q's, something has to be known regarding the time relation of the risks. A simplifying assumption, which is frequently reasonable, is to suppose that each instantaneous risk, which is called the "force of mortality" in actuarial texts, is constant over the period of observation. If l_t is the number of survivors at time t, then

$$-\frac{dl_t}{l_t dt} = -\frac{d\ln l_t}{dt}$$

is the instantaneous risk. I will use β's to represent the instantaneous risks, and shift to the example of dealing with two mortality rates, q_1 the net mortality from some specified disease, and q_2 the net mortality rate from all other diseases than 1, taken together and considered as a single risk. Then the net probability of death from the respective causes at time <t is given by

$$q_{1_t} = 1 - e^{-\beta_1 t} \tag{8}$$

$$q_{2_t} = 1 - e^{-\beta_2 t} \tag{9}$$

where β_1 is the instantaneous risk for net death risk 1, β_2 is the instantaneous risk for net risk 2, and t is the time measured from t = 0.

From (8), (9) we have the corresponding net probability of survival to time t

(10) $$p_{1_t} = 1 - q_{1_t} = e^{-\beta_1 t}$$

(11) $$p_{2_t} = 1 - q_{2_t} = e^{-\beta_2 t}.$$

The probability of survival to time t, with both risks operating together is the product of (10) (11)

(12) $$P_t = e^{-(\beta_1 + \beta_2)t} = e^{-\beta t}$$

and the probability of dying at time $<t$ is

(13) $$Q_t = 1 - P_t = 1 - e^{-\beta t}$$

where $\beta = \beta_1 + \beta_2$.

The formulas (10), (11), (12) represent "survival functions" in the context of actuarial discussions.

Without loss of generality, we can consider the period of observation as from $t = 0$ to $t = 1$.

The proportion of persons dying from cause 1 over the unit period, say a year, from $t = 0$ to $t = 1$ is the crude death rate from cause 1. It is

(14) $$Q_1 = \int_0^1 e^{-\beta t} \beta_1 \, dt = \frac{\beta_1}{\beta}\left(1 - e^{-\beta}\right) = \frac{\beta_1}{\beta} Q$$

and similarly for cause 2

(15) $$Q_2 = \frac{\beta_2}{\beta}\left(1 - e^{-\beta}\right) = \frac{\beta_2}{\beta} Q$$

and for total deaths from all causes

(16) $$Q = Q_1 + Q_2 = 1 - e^{-\beta}$$

and the probability of survival to the end of the period is

(17) $$P = 1 - Q = e^{-\beta}.$$

The net death rates over the unit period are

(18) $$q_1 = 1 - p_1 = 1 - e^{-\beta_1}$$

(19) $$q_2 = 1 - p_2 = 1 - e^{-\beta_2}.$$

Now, we observe the crude rates Q_1, Q_2 and $Q = Q_1 + Q_2$; we wish the net rates q_1, q_2. These can be derived directly from (14), (15), (18), (19), and are given by

(20) $$\ln (1 - q_1) = -\beta_1 = \frac{Q_1}{Q} \ln (1 - Q)$$

(21) $$\ln (1 - q_2) = -\beta_2 = \frac{Q_2}{Q} \ln (1 - Q) .$$

MAXIMUM LIKELIHOOD FREQUENCY ESTIMATE. The development of the formulas for obtaining the net rates q_1, q_2 just given in (20), (21) is what is sometimes called "deterministic." We simply solved algebraically for the q's, having written down the equations representing the assumptions. If we stop to think a moment, in making these solutions we said we knew the crude rates Q_1, Q_2. But how are we to know them? We assume that we have observed them -- the Q's represent the "observed" rates which are computed by dividing deaths by N. But from a statistical view, if the numbers N on which these observed rates are based are moderate or small, we do not "know" the Q's -- these are only estimates. I will now consider the problem from the stochastic view, and specifically will develop the maximum likelihood estimates and their variances.

N individuals are observed over the unit period from $t = 0$ to $t = 1$. We observed d deaths, d_1 from cause 1, d_2 from cause 2, and $s = N - d_1 - d_2$ survivors to the end of the period. First, it will be convenient to estimate $\beta = \beta_1 + \beta_2$. Since the crude probability of death is $(1 - e^{-\beta})$, and of survival it is $e^{-\beta}$, the probability of the sample is proportional to

(22) $$\phi = (1 - e^{-\beta})^d \, e^{-\beta s} .$$

From (22) we derive the maximum likelihood estimate and its variance in the standard way.

(23) $$\hat{\beta} = \ln (N/s)$$

(24) $$\sigma^2_{\hat{\beta}} = \frac{1 - e^{-\beta}}{N e^{-\beta}}$$

To derive the estimates of β_1 and β_2 we write the probability of the sample in terms of d_1 and d_2. It will be remembered that the crude probability of death from cause 1 is $\frac{\beta_1}{\beta}(1 - e^{-\beta})$, and from cause 2 it is $\frac{\beta_2}{\beta}(1 - e^{-\beta})$, and the probability of survival to the end of the period is $e^{-\beta}$. The probability of the sample is then proportional to

(25) $$\phi = \left[\frac{\beta_1}{\beta}(1 - e^{-\beta})\right]^{d_1}\left[\frac{\beta_2}{\beta}(1 - e^{-\beta})\right]^{d_2} e^{-\beta s}$$

and from this we obtain

(26) $$\hat{\beta}_1 = \frac{d_1}{d} \ln (N/s) = \frac{d_1}{d}\hat{\beta}$$

(27) $$\hat{\beta}_2 = \frac{d_2}{d} \ln (N/s) = \frac{d_2}{d}\hat{\beta}$$

(28) $$\sigma^2_{\hat{\beta}_1} = 1/N\left[\frac{\beta_1 \beta_2}{(1 - e^{-\beta})} + \frac{\beta_1^2 (1 - e^{-\beta})}{\beta^2 e^{-\beta}}\right]$$

(29) $$\sigma^2_{\hat{\beta}_2} = 1/N\left[\frac{\beta_1 \beta_2}{(1 - e^{-\beta})} + \frac{\beta_2^2 (1 - e^{-\beta})}{\beta^2 e^{-\beta}}\right].$$

We obtain the estimate of the q's from the estimates of the β's by the corresponding relation to the parameters, for instance

$$q_1 = 1 - e^{-\beta_1}$$

$$\text{var. } q_1 = (1 - q_1)^2 \text{ var. } \beta_1 .$$

If these maximum likelihood estimates which I call the "frequency estimates" are examined, it will be found that, in effect, they are the same as the estimates derived on a deterministic basis, since in that case we take the crude probability Q as given by the corresponding observed relative frequency d/N. However, with the development of the maximum likelihood estimate, we have also the large sample variance.

MAXIMUM LIKELIHOOD TIME ESTIMATES. In developing the maximum likelihood frequency estimate as just completed, we took into account only the number of deaths from each cause in the unit period. We did not use any information on the times of the deaths. But if the survival functions are of assumed form, these times should help us estimate the parameters β. I will now develop the maximum likelihood estimates using the times of death. The d_1 deaths from cause 1 have been observed at times t_1, the d_2 deaths at times t_2.

It will be convenient, as before, first to estimate $\beta = \beta_1 + \beta_2$. For a death at time t among the $d = d_1 + d_2$ deaths, the probability is $\beta e^{-\beta t}$, and for a survivor to the end of the period, the probability is $e^{-\beta}$. For the total sample the probability is proportional to

$$\phi = \beta^{d} e^{-\beta\Sigma t} e^{-\beta s} \quad . \tag{30}$$

From this we derive the maximum likelihood estimate and its asymptotic variance [4], [5], [9]

$$\hat{\beta} = \frac{d}{\Sigma t + s} \tag{31}$$

$$\sigma^2_{\hat{\beta}} = \frac{\beta^2}{N\,(1 - e^{-\beta})} \quad . \tag{32}$$

For the estimate of β_1 and β_2, we write the probability of the observations of the numbers and times of death from cause 1 and cause 2, and the survivors to the end of the period. Then the probability of the observations is proportional to

$$\phi = \beta_1^{d_1} e^{-\beta\Sigma t_1} \beta_2^{d_2} e^{-\beta\Sigma t_2} e^{-\beta s} \tag{33}$$

where $\beta = \beta_1 + \beta_2$.

From (33) we derive the maximum likelihood estimates and their asymptotic variances.

$$\hat{\beta}_1 = \frac{d_1}{\Sigma t + s} , \quad \hat{\beta} = \frac{d_2}{\Sigma t + s} \tag{34}$$

where $\Sigma t = \Sigma t_1 + \Sigma t_2$

(35) $$\sigma^2_{\hat{\beta}_1} = \frac{\beta_1^2 + \beta_1 \beta_2}{N(1 - e^{-\beta})}$$

(36) $$\sigma^2_{\hat{\beta}_2} = \frac{\beta_2^2 + \beta_1 \beta_2}{N(1 - e^{-\beta})}$$

COMPARISON OF THE FREQUENCY AND TIME ESTIMATES. Two sets of maximum likelihood estimates have been developed, one based on the observed frequencies of death from each cause, the other using also the times of these deaths. Presumably the time estimates, which use more "information," are better, and this should be reflected in a smaller variance of the time estimates. I shall compare the variances of the frequency and time estimates of β.

It is clear on inspection that the frequency estimate cannot be good for large β, $Q \to 1$,

$$\hat{\beta} = \ln (N/s) .$$

If Q is nearly unity the probability that s = 0, for even fairly large N, will not be small, and for all samples with s = 0, the frequency estimate of β is not determinable. In table 1 are shown the relative variances of the two estimates for different values of Q. It is seen that for small Q = .05 the variance of the time estimate is virtually equal to that of the frequency estimate. For $Q \leq 0.6$, the relative efficiency is greater than 0.9. Only with $Q > 0.9$ does the efficiency fall below 0.5. Since the frequency estimate requires only the number of deaths and not their times, and is easier to compute than the time estimate, it may be found satisfactory for use, except with very large Q.

MEASURE OF THE MORTAL EFFECT OF SMOKING. The ideas and formulas developed above are applicable to the analysis of the data of "prospective" studies into the relation of smoking and lung cancer. As a matter of fact Dr. Mindel Sheps [11], on the basis of a heuristic approach involving the notion of "exposed to risk," derived a maximum likelihood estimae which is identical with that developed here as the maximum likelihood frequency estimate of the net probability of death, from all causes, attributable to smoking. I shall consider the problem more in detail, in terms of the development I have outlined, particularly in respect of deaths from specific causes.

Consider deaths as segregated in two classes: those due to (1) some specific disease, for which I take lung cancer as an example, and (2) all other causes taken together. Non-smokers are subject to deaths from "natural causes." Smokers also are subject to death from natural causes, but we assume that, in addition, they are subject to deaths from lung cancer caused by specific carcinogens Y, and from other diseases caused by substances X, these substances Y and X being contained in tobacco smoke. We assume that these causes act independently, and that the net probability of death, at time $< t$ $(0 \leq t \leq 1)$ in a unit period are given by

(37) $$q'_{t_1} = 1 - e^{-\beta'_1 t}$$

(38) $$q'_{t_2} = 1 - e^{-\beta'_2 t}$$

(39) $$q_{t_1} = 1 - e^{-\beta_1 t}$$

(40) $$q_{t_2} = 1 - e^{-\beta_2 t}$$

where q'_{t_1}, q'_{t_2} refer to net probabilities of death due to natural causes, from lung cancer and other diseases respectively, and q_{t_1}, q_{t_2} refer to death from lung cancer and from other diseases caused respectively by substances Y and substances X contained in tobacco smoke.

The corresponding observed crude probabilities of death are then

(41) $$Q'_{t_1} = \frac{\beta'_1}{\beta'}\left(1 - e^{-\beta' t}\right)$$

(42) $$Q'_{t_2} = \frac{\beta'_2}{\beta'}\left(1 - e^{-\beta' t}\right)$$

(43) $$Q'_t = 1 - e^{-\beta' t}$$

(44) $$Q_{t_1} = \frac{\beta_{11}}{\beta_T}\left(1 - e^{-\beta_T t}\right)$$

(45) $$Q_{t_2} = \frac{\beta_{22}}{\beta_T}\left(1 - e^{-\beta_T t}\right)$$

(46) $$Q_t = 1 - e^{-\beta_T t}$$

where

$$\beta' = \beta_1' + \beta_2'$$

$$\beta_{11} = \beta_1' + \beta_1 \quad ; \beta_{22} = \beta_2' + \beta_2$$

$$\beta_T = \beta_{11} + \beta_{22} = \beta_1' + \beta_1 + \beta_2' + \beta_2 \ .$$

N' nonsmokers have been observed, of whom d_1' have died from lung cancer at times t_1', and d_2' have died from other diseases at times t_2', while $s' = N' - d'_1 - d'_2$ have survived to the end of the period. We wish the maximum likelihood estimates of β'_1, β'_2, β_1, β_2, the corresponding net probabilities of death from lung cancer and from other diseases, attributable to natural causes, and attributable to cancer. We can derive these as before by writing out the probability of the total set of observations, including those on the nonsmokers and those on the smokers. However, the estimates may be had directly from the formulas already developed.

For the nonsmokers the parameters β'_1, β'_2, β', and the corresponding q's, and Q's which are functions of the β's are obtained directly from the formulas given, since these are the parameters involved in the exponential functions representing the probabilities of death among the nonsmokers. So far as the smokers are concerned, considering lung cancer, the deaths are due to (1) natural causes and (2) substances Y. We remember that in the exponential model the net risks are additive, so the exponential parameter of the smokers representing the risk for lung cancer is $\beta_{11} = \beta'_1 + \beta_1$. And similarly the exponential parameter representing the risk of death from other diseases among the smokers is $\beta_{22} = \beta'_2 + \beta_2$ as presented in (44), (45). Then β_{11} and β_{22} can be estimated from the observations on the smokers. Now, the observations on the nonsmokers and on the smokers are independent since they are made on different samples. So we obtain the estimate of β_1 and β_2 by subtraction

$$\hat{\beta}_1 = \hat{\beta}_{11} - \hat{\beta}_1' \tag{47}$$

$$\hat{\beta}_2 = \hat{\beta}_{22} - \hat{\beta}_2' \ . \tag{48}$$

Similarly, since the estimates are independent, the variances are obtained as the sum of the variances of $\hat{\beta}_1'$ and $\hat{\beta}_{11}$. All the other estimates which are required are functions of the β's which have been estimated, and may be obtained by using the formulas for estimating functions.

The estimates of all the parameters involved in the analysis, with their variances, will be presented in a paper to be published later.* The chief parameter of interest here is the net probability of death due to a specified disease, here taken as lung cancer. I shall only write down the estimates for this parameter, which, it will be remembered, is symbolized by q_1.

(49) The frequency estimate is given most simply by,

$$\ln (1 - \hat{q}_1) = \frac{d_1}{d} \ln (s/N) - \frac{d_1'}{d'} \ln (s'/N').$$

The time estimate is given by

$$\ln (1 - \hat{q}_1) = \frac{d_1'}{\Sigma t' + s'} - \frac{d_1}{\Sigma t + s} . \tag{50}$$

I take as an example of the application of the derived formulas, some data from the prospective study sponsored by the American Cancer Society and reported by Hammond and Horn [6], [7], [8]. Some 200,000 men in the age range 50 to 70 years were interviewed and a statement obtained from each as to his smoking habits. Periodically, inquiry was made and it was ascertained when any individual had died, and the time and cause of death as stated on the death certificate were recorded. A report was made based on the status of each individual as of 44 months after the initial inquiry. In table 2 are shown the essential data for the group of men 60-65 years of age at the time of the original inquiry. The binomial estimates of the probability of death in the 44 month period of follow-up are shown, for the nonsmokers and for the smokers, for each of four categories of cause of death, namely cancer of the lung, other cancer, coronary artery disease, and other diseases, as well as for death from all diseases. In the last 3 columns are shown three indices of the effect of smoking in increasing the probability of death from each of the categorized causes. The first of these indices is the estimate of the net probability of death from the respective causes, using the frequency maximum likelihood estimate. The second is the simple difference of the probabilities of death of smokers and of nonsmokers. The third column gives the so-called mortality ratio, which here is the ratio of the probability of death among smokers to the probability among nonsmokers.

If we use the net probability of death as the measure of the effect of smoking in respect of a cause of death, we see that, among the four categorized causes of death, smoking has the greatest effect in increasing deaths from coronary heart disease, the next greatest with diseases in the class "other diseases," the next from cancer other than lung cancer, and the least from lung cancer. If we use the simple difference of the probabilities of death, shown in the next column, we reach essentially

* Jointly with Dr. Lila Elveback.

the same conclusion. This is not surprising, since it is easy to show that if the probabilities of death Q, q are small, the net probabilities are given with close approximation by the simple difference of the probabilities of smokers and nonsmokers. If we use the "mortality ratio" shown in the last column, a quite different impression is obtained. We see that this index is 9.7 for lung cancer, while it is less than 2 for each of the other categories. In at least one important report from the United States Public Health Service [3], a ratio of less than 2 was considered as not even worth reporting as physically significant, and if this view is applied to the data of the table presented, it would only be said that these data show that smoking causes lung cancer. In an official statement on smoking by the Surgeon General [2] of the United States Public Health Service -- which depends largely upon the study represented by table 1, and other studies showing similar results -- only lung cancer is mentioned!

Which interpretation is valid -- that smoking is associated with death from all classes of disease, and chiefly from diseases other than lung cancer, or that drawn from the mortality ratio, which indicates a great effect on lung cancer and only a relatively small and negligible effect on any other disease?

The use of the mortality ratio has been criticized on a number of general grounds by Sheps [12] and by me [1] and it seems to the point to summarize some of them.

> If N animals are exposed to smoking and d_s die, while among N control animals d_o die, then the mortality ratio divides d_s by d_o to obtain a measure of the mortality due to smoking. In the conception of a death rate that places the number of "exposed to risk" in the denominator, this enumerates the dead controls as the "exposed to risk," and seems to imply that it is only those who are already dead from natural causes that can be killed by smoking! This has prompted Sheps ironically to title her article "Shall We Count the Living or the Dead?"
>
> It is arbitrary to use the ratio of mortality rates rather than the ratio of survival rates, and each gives a very different answer to the questions of the problem in hand.
>
> If a mortal drug were tried with controls, using the mortality ratio, it would appear to have a larger effect in a season when the natural mortality was low, than when it was tried in a season during which the natural mortality was high -- even if the actual effect of the drug was unaltered.
>
> Use of the ratio makes a small increase of deaths from a disease in the smoking group appear inordinately large if the natural mortality from that disease is small, and reduces to absurdity if the natural mortality is zero.

Yet there has been a great use of the mortality ratio in the studies referred to, with consequent emphasis on lung cancer. Now, the interpretation of the biologic significance of these statistical findings turns critically on how they are reported. If it is reported that smoking causes many diseases -- including such diseases as cancer of the prostate, for which no physical explanation is at hand -- it may be considered that the studies "prove too much," and that they are spurious, arising possibly from some unrepresentativeness in the sample. Or, if they are not spurious, they will perhaps be interpreted as reflecting a constitutional difference between nonsmokers and smokers rather than as supporting the theory that smoking causes lung cancer. But the general public, and also statisticians generally, have received the impression that all the statistical studies show is that smoking causes lung cancer. The public has not been told with anything like equal clarity, that smoking, in these statistical studies, seems to cause all classes of disease -- lung cancer only to the extent of about 15 per cent of the total. The statistical basis of this emphasis on lung cancer seems to be the use of mortality ratios, instead of net rates, or difference of death rates, to measure the putative mortal effect of smoking. This is the reason for linking the present paper on exponential competing risks to the statistical study of smoking and lung cancer.

Table 1

Comparison of Variance

Time estimate and frequency estimate

Q	N x Variance: Time Estimate	N x Variance: Frequency Estimate	Relative Efficiency
.001	.0010	.0010	1.000
.01	.0101	.0101	1.000
.05	.0526	.0526	1.000
.10	.1110	.1111	.999
.50	.9609	1.0000	.961
.60	1.3993	1.5000	.933
.90	5.8910	9.0000	.655
.95	8.9744	19.0000	.472
.99	21.4218	99.0000	.216

Deaths in 44 months. Age at beginning, 60-64 years

Cause of death	Nonsmokers* 20,278		Smokers** 21,594		Measure of effect of smoking		
	Deaths		Deaths		Net prob. due to smoking	Diff. of prob.	Mort. ratio
	No.	Prob.	No.	Prob.			
Lung cancer	10	.00049	103	.00477	.00449	.00428	9.7
Other cancer	218	.01075	325	.01505	.00469	.00430	1.4
Coronary disease	552	.02722	921	.04265	.01653	.01543	1.6
Other diseases	486	.02397	667	.03089	.00765	.00692	1.3
Total: All causes	1266	.06243	2016	.09336	.03303	.03093	1.5

* Nonsmokers = never smoked cigarettes regularly.

** Smokers = all regular cigarette smokers.

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ARMY RESEARCH AND DEVELOPMENT

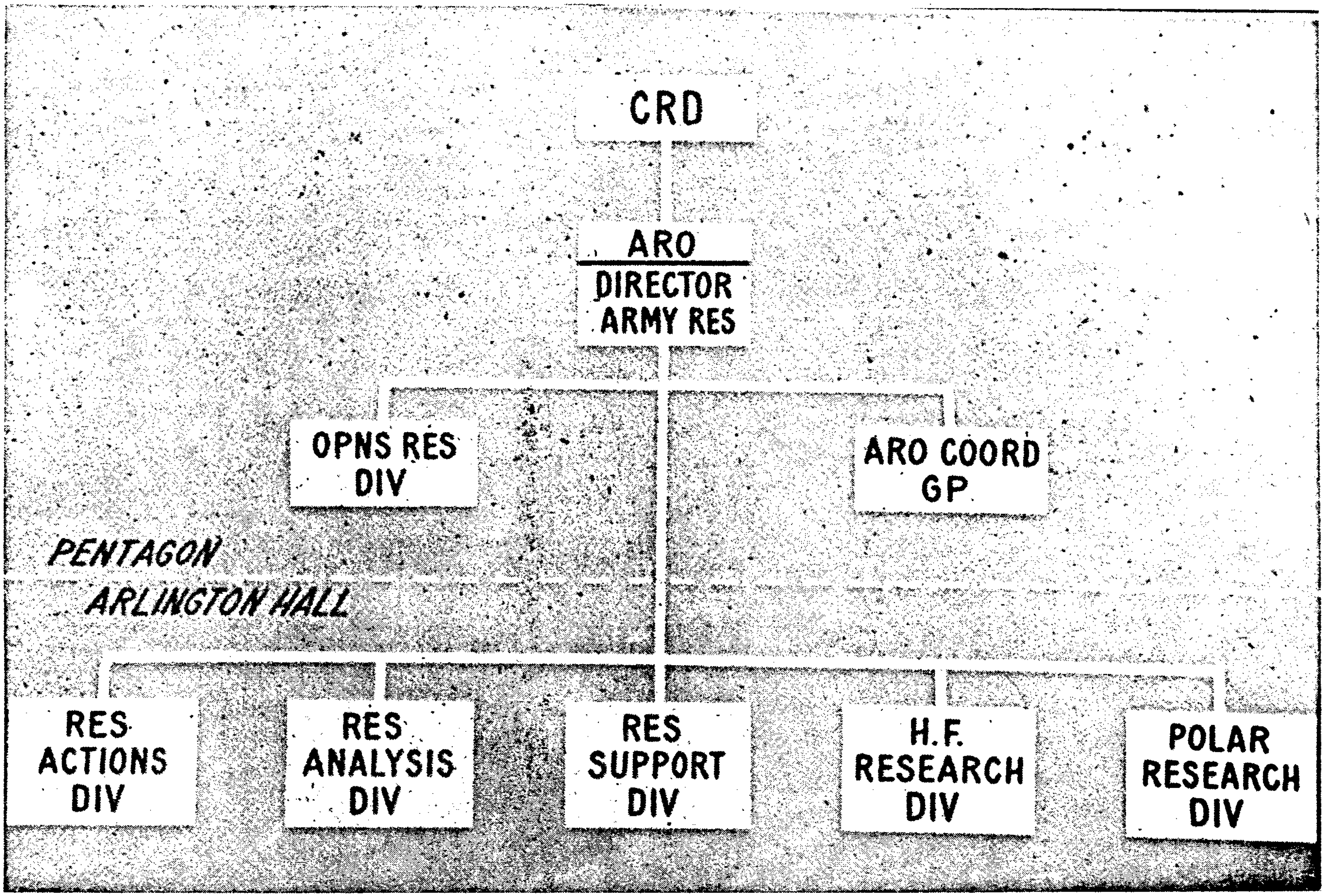
Richard A. Weiss
Army Research Office

Ladies and Gentlemen: What I want to talk about is Army Research and Development in its general aspects. Even though many of us are in the Army, it has been my experience that as members of one Technical Service, we tend to forget that there are other programs than our own. As a matter of fact, each Technical Service, in its own area, has responsibility for work of major importance in the research and development field. I thought if I could go through the program of the Army in a rather rapid fashion, it would give you some appreciation of the scope and possibly the depth of the programs being worked on by the seven Technical Services and their impact on the civilian economy.

We start with the organization of the Army. The organization of the Department of Defense might be likened to an onion, one of the many layers of which is the Army. The Chief of Research and Development is responsible for the planning and direction of research and development in the Army. He does this through three directorates: The Director of Developments, who is responsible for communications electronics, surveillance, the development of combat equipment, Army aviation, and the many developments necessary for the support of the ground soldier; the Director of Special Weapons, who is responsible for such areas as nuclear power, the nuclear aspects of missiles, and generally, with the overall weapons program of the Army; and the Director of Army Research, who has responsibility for monitoring the entire research program of the Army which is extensive and diverse. The Army Research Office, which is the operating element of the Directorate of Army Research, is, as you know, a rather newly formed office. It is in the process of being staffed and hopes to be in such a position as to present a sound scientific Army position to the outside scientific community and also do the job that it has to do to defend the support of basic research at the Defense level and in the Army Staff itself.

Slide 1

ARO Organization

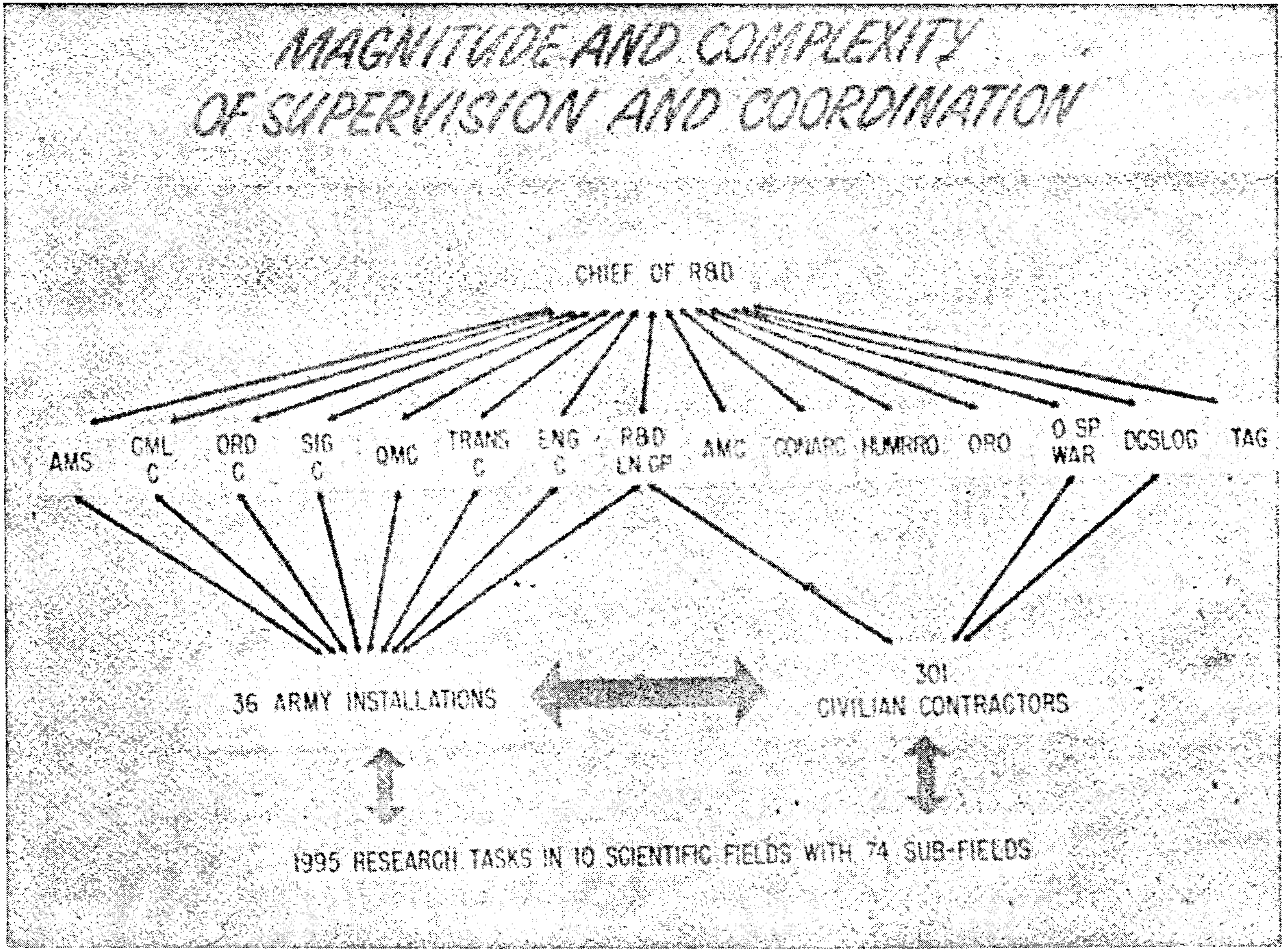


Slide 1

The first slide is a representation of the present Army Research Office organization. As you see, there is an Operations Research Division. This division is responsible for special studies cutting across much of the Army's overall mission and, particularly, those relating to research in the Army's operational problems. The Research Support Division has responsibitity for activities relating to scientific manpower, scientific information, and symposia and conferences of a scientific nature. It has just completed necessary staffing on a tri-service grants program. Human Factors Division is concerned with the problems of training and leadership and the relationship of the soldier to the machine. The three scientific divisions - Environmental Sciences Division, Life Sciences Division, and Physical Sciences Division - are composed of civilian and military scientists, all specialists in various scientific disciplines. In their particular fields, they analyze the program to determine gaps, determine the proper program balance, and develop policies effecting the improvement of the environment of the scientists in various laboratories and arsenals in the Army.

Slide 2

Magnitude and Complexity
of CRD



Slide 2

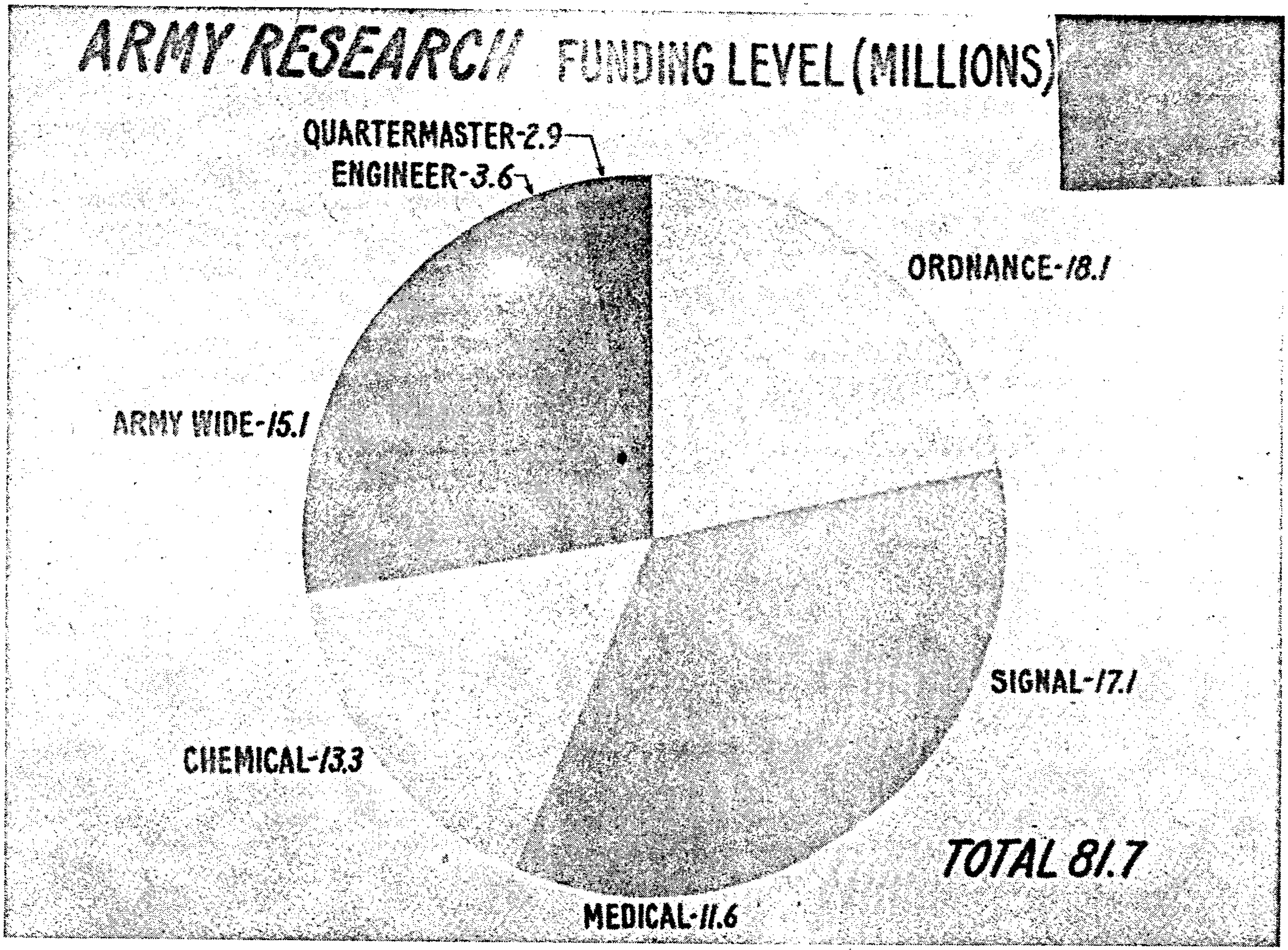
I don't know whether this can be seen, but this is an idea of the magnitude of supervision and coordination that the Chief of Research and Development has. The first seven blocks on the left are the seven Technical Services - Army Medical Service, Chemical Corps, Ordnance Corps, Signal Corps, Quartermaster Corps, Transportation Corps, and the Corps of Engineers. There is a Research and Development liaison group in Frankfort, Germany, carrying out the support of sciences in European communities. There is the Army Mathematics Center at the University of Wisconsin; R&D support for the Continental Command at Fort Monroe; The Human Factors Research Office at George Washington University; The Operations Research Office at Johns Hopkins University; R&D support for a division of Special Warfare; and R&D support of operations research for the Deputy Chief of Staff for Logistics.

These are the areas where the Chief of R&D has to provide funds for the support of the work that is going on. It is his responsibility to get the funds to carry out the mission and to provide support for the scientific staff. There are 38 Army installations where the research is being done. This is not generally known by most people in the Army. There are 19 government and over 400 civilian contractors, and as of today there are about 2400 research tasks in the various scientific disciplines covering something in the order of 74 sub-fields.

Now, a little bit about funding. The Chief of R&D has the problem of maintaining a balanced program, not only with the logistics of production but, once having got his share, maintaining a balanced program between the development program and the research program. And this is a rather trying task because each of these areas makes its own rather severe demands. Regarding the total funds in 1960: There is, roughly, a billion dollar budget - half a billion dollars in research and development and another half billion in test and evaluation. It will be divided, roughly, in the following fashion:

Slide 3

Funding (Pie View)



Slide 3

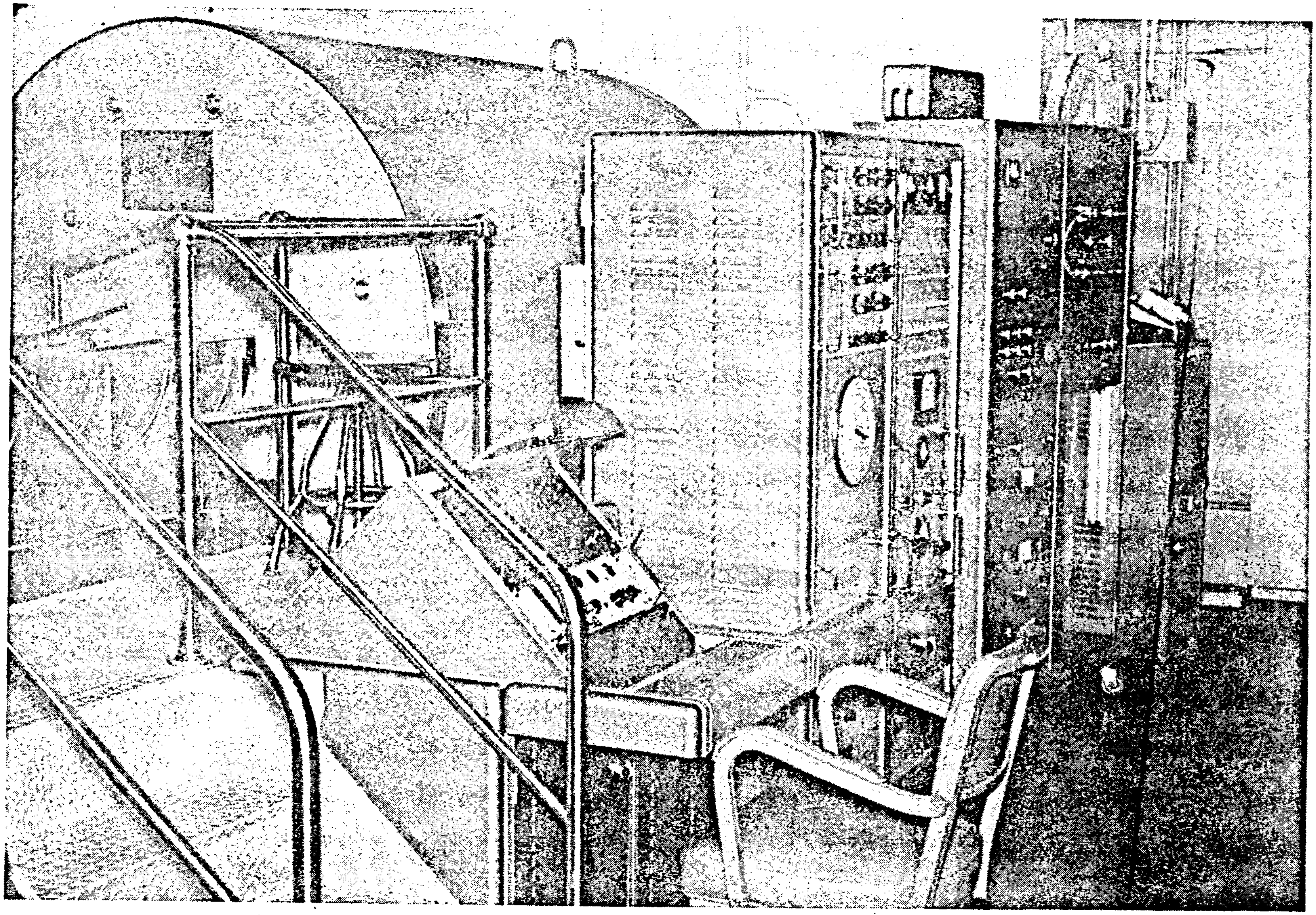
This slide shows a pie view of the allocation of funds among the seven Technical Services. You see Army-wide - $15.1 million - that has to do with a rather large program that goes across the entire board. Here the Office of Ordnance Research, the Army Mathematics Center, the European Research Office, and the new office in Japan are supported. Now, going around, you can see the relative proportions. This shows a total of $81.7 million in a prior year, but the program still balances up approximately the same, and the total research support, which is in the order of $130 million in 1960, will be divided among the Technical Services approximately according to the percentages shown here. Now, basic research is about 35 million dollars, which is of the order of one-fourth of the total Research and Development Program of $130 million. So I think that a ratio of one dollar for every four R&D dollars in support of research is a pretty healthy indication of the support of research that the Army is giving.

I would like to point out that the total contracts to non-government installations by all the seven Technical Services is in the order of 4,000. I think anyone would agree that in the research field alone, and I am not talking development or test and evaluation, the dollars that the Army spends certainly make a major impact on the total economy of the country. Obviously, the Navy and the Air Force do the same, the sum of money that is spent is considerable, and I would like to spend a little bit of time later indicating some of the things that have come out of the program.

Now, a few things about the Army installations.

Slide 4

Whole Body Radiation Counter



Slide 4

This is a slide of the whole body radiation counter at the Walter Reed Army Institute for Medical Research. It is possible to put a man inside so that the radiation emitted by virtue of any radioactive material that is in his body can be counted by a bank of scintillation counters. As a matter of fact, it is the only one, I believe, in the country, and much good research has been done at Walter Reed in this field.

Slide 5
(on following page)

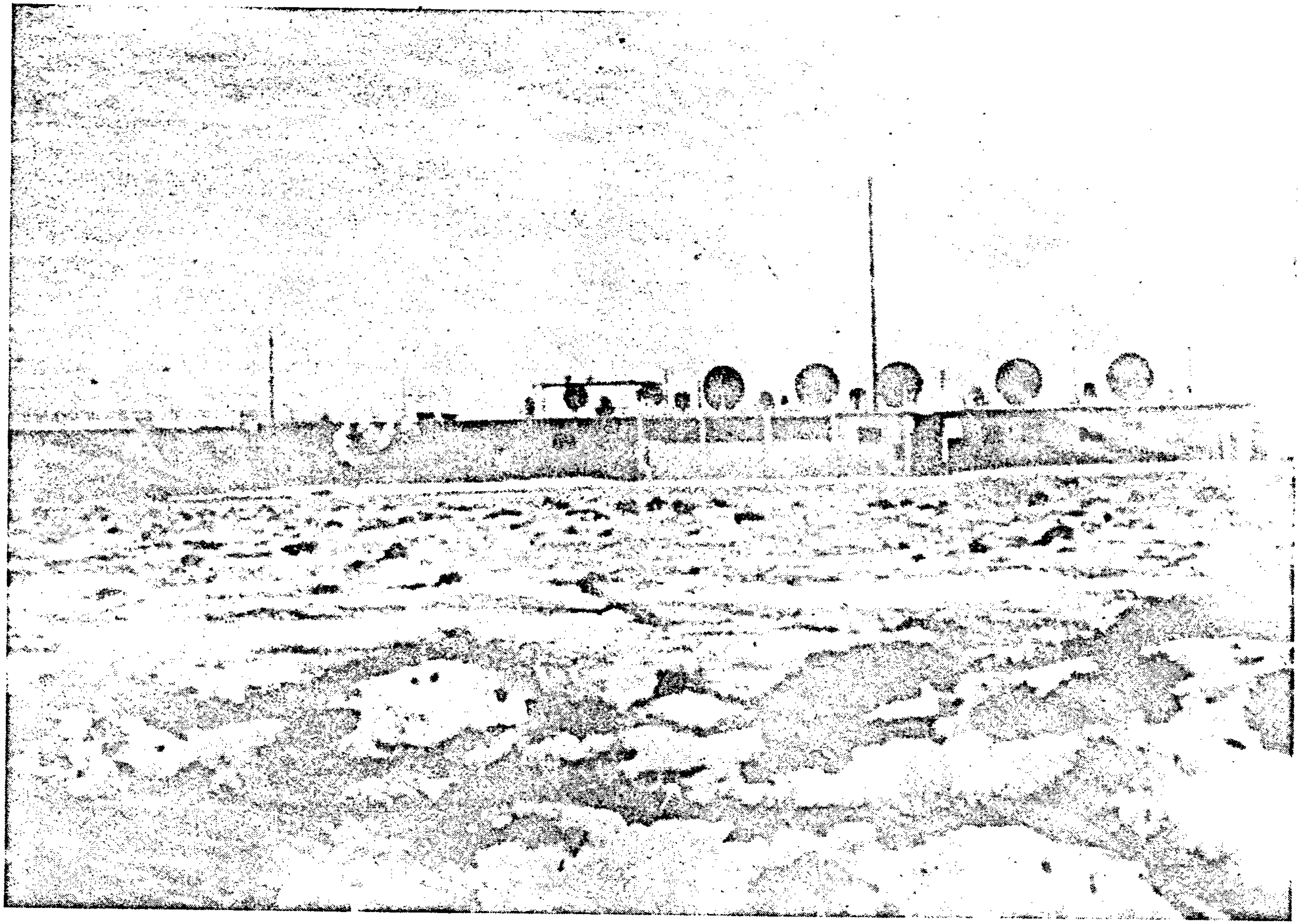
Next is a slide of the White Sands Proving Ground which many of you will recognize. This is the Range Control Station for continuous plot of trajectories; a great amount of information is ground out here during the test flights.

Slide 6

The next slide shows an engine on a static test stand.

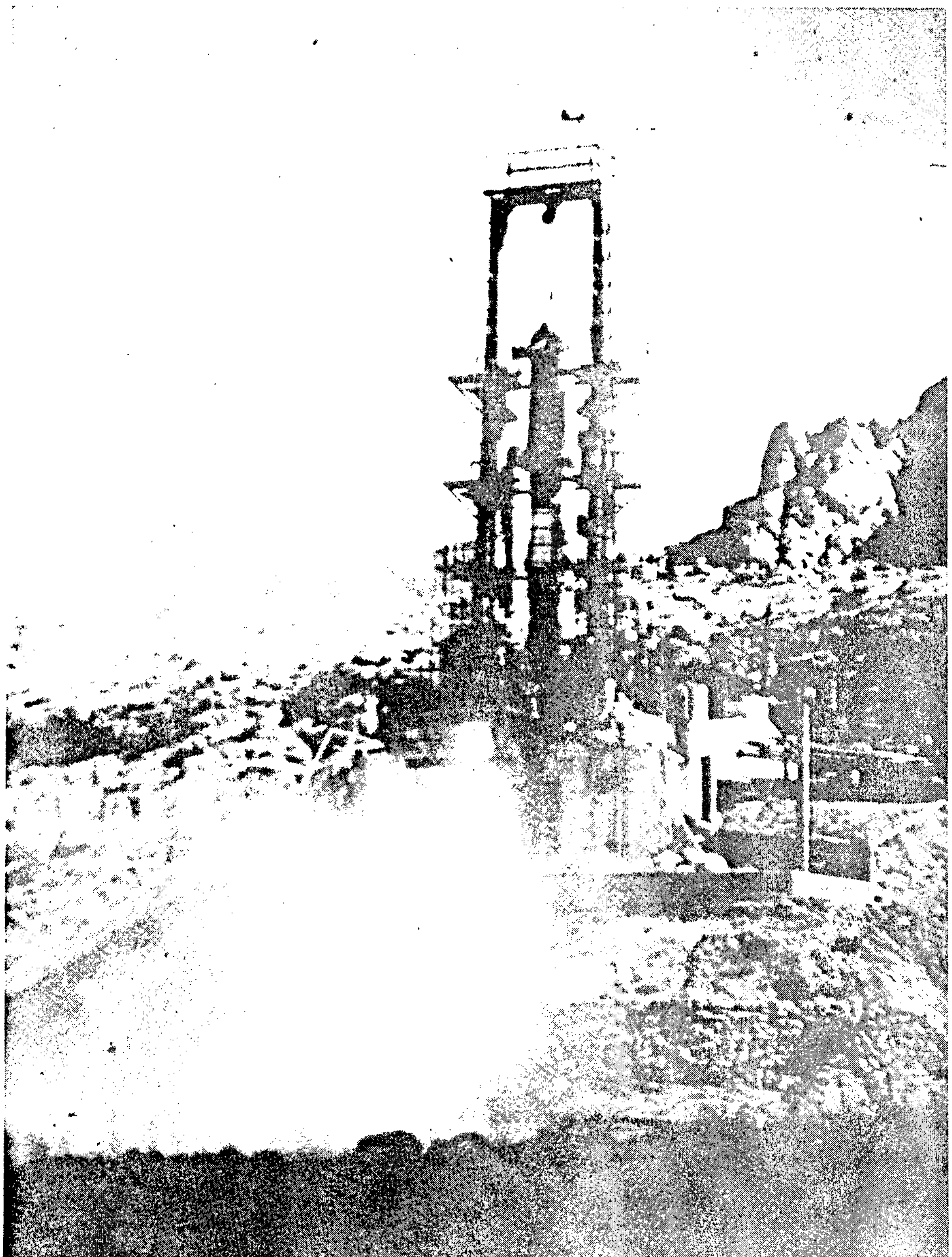
Slide 5

White Sands Proving Ground



Slide 6

Engine on Static Test Stand



Slide 7

Thule, Greenland

Slide 7

The following slide was taken in Thule, Greenland. Here there are something like nine disciplines covered in the 50 research tasks that are being undertaken by all the Technical Services. This is the area where the Engineers and the Transportation Corps, particularly, are engaged in a major program in determining how to survive and come to terms with the environment in these latitudes. Some rather remarkable and unique types of operations have been carried out by the Engineers under ice. Here ice is handled as one would cut stone from a quarry and ice construction is carried on underground. Laboratories have been established and experiments on ice and its characteristics are carried out.

Slide 8

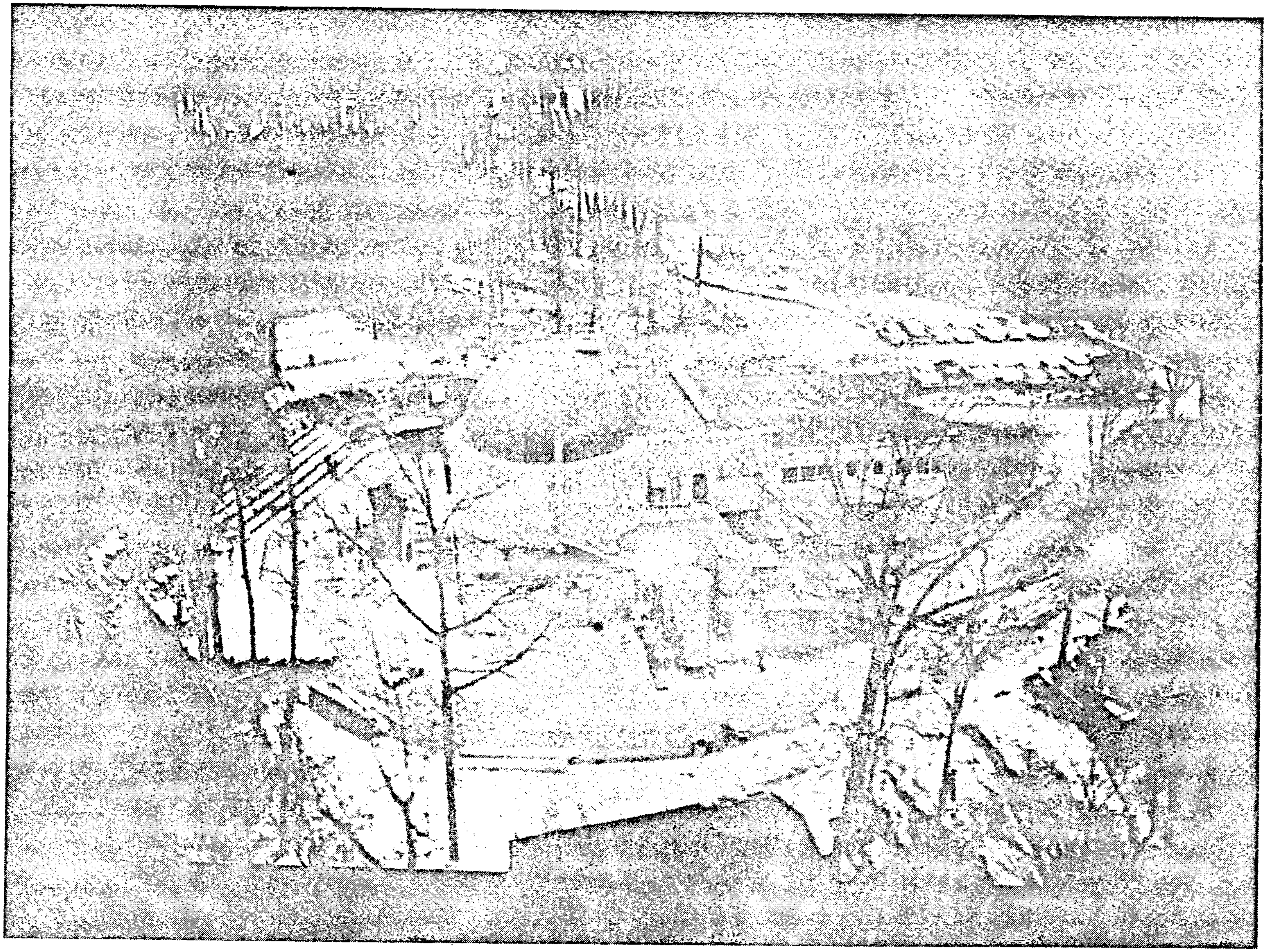
This is something which will be recognized. It is the Army's packaged power reactor at Fort Belvoir, Virginia. The general feeling is that some rather remarkable things can be done in isolated areas with a facility of this kind. It is a 2-megowatt, thermally measured power apparatus, and is a prototype for others being built.

Slide 9

The next slide is of the solar furnace just recently installed at the Quartermaster Corps Natick Laboratory. The little white house in the center is the place where the beam is focused. There is a plane of mirror which tracks the sun and directs the parallel rays into a convex focusing mirror. Temperatures up to 3500 F. are achievable with fluxes of the order of 75 calories per sp. cm.

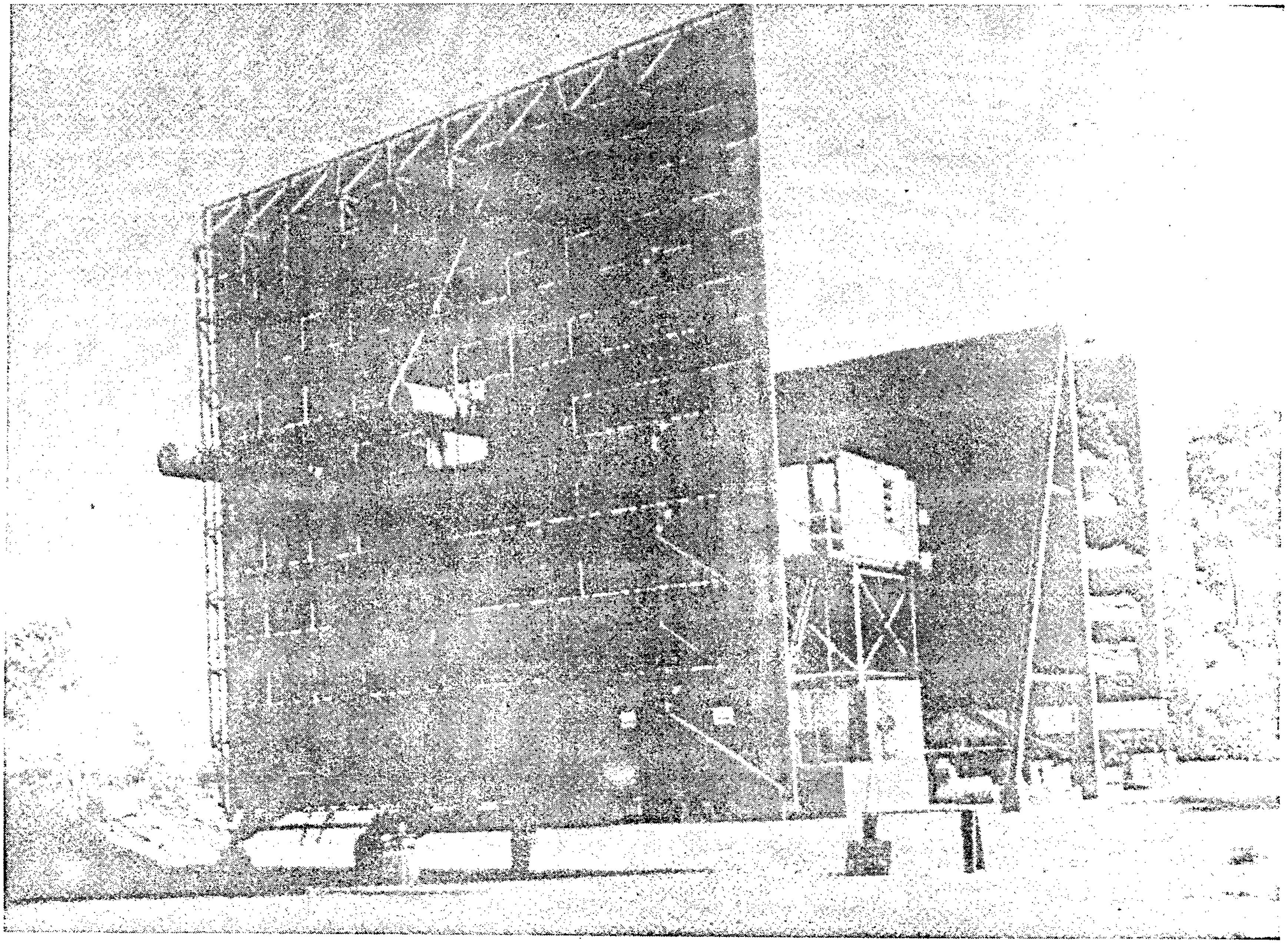
Slide 8

Power Reactor, Fort Belvoir



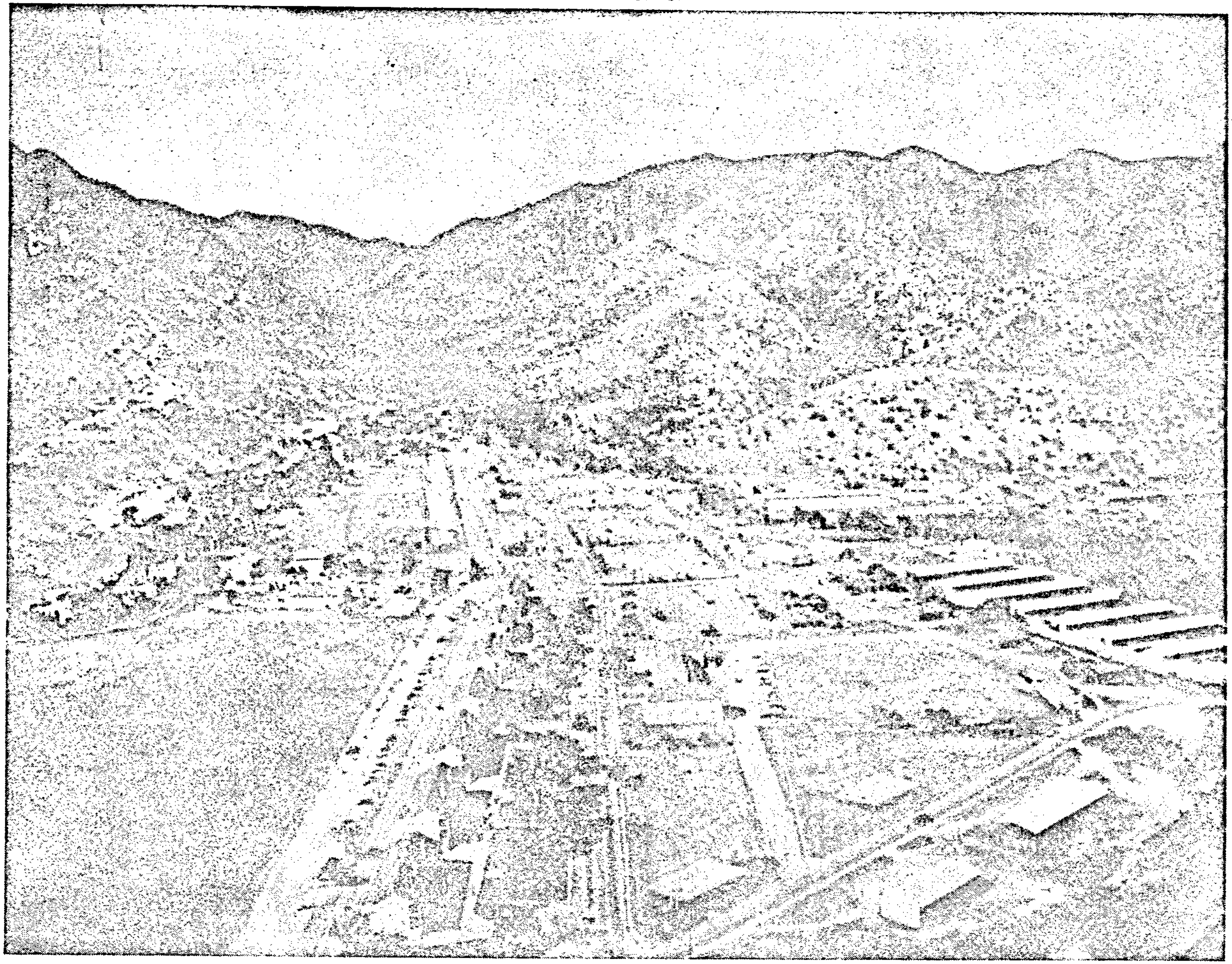
Slide 9

Solar Furnace



Slide 10

Fort Huachuca



Slide 10

This is a slide of a rather famous place. It is the old cavalry post at Fort Huachuca, in Arizona. It is now run by the Signal Corps and is an electronic proving ground where a great deal of testing is being done on surveillance devices, on the effect of counter-measures and general testing and evaluation in the field of communications for combat purposes. The red-roofed buildings are the old buildings that were there when the post was first established many years ago.

These are only a few of the number of installations that the Army has, and as a matter of fact, when one visits and sees the modern scientific and technical equipment available in the laboratories, one really has a great respect for the diversity and depth of Army science.

Now a few things about the accomplishments that the Army has been able to achieve which are of value to the civilian economy. Certainly World War II gave impetus to the aircraft industry and gave the chemical industry its greatest change to produce. It also ushered in the electronic age and the nuclear age. Following these, we now have the space age. Certainly one can expect, as time goes on, that the Department of Defense, with its three services, will be making other important contributions to our economy.

The Quartermaster Corps has done a major job in the processing and packaging of foods, much of which finds its way into the civilian economy. Pasteurization, dehydration, development of balanced diets for large groups, and minimum weight packaged material represent their area of contribution.

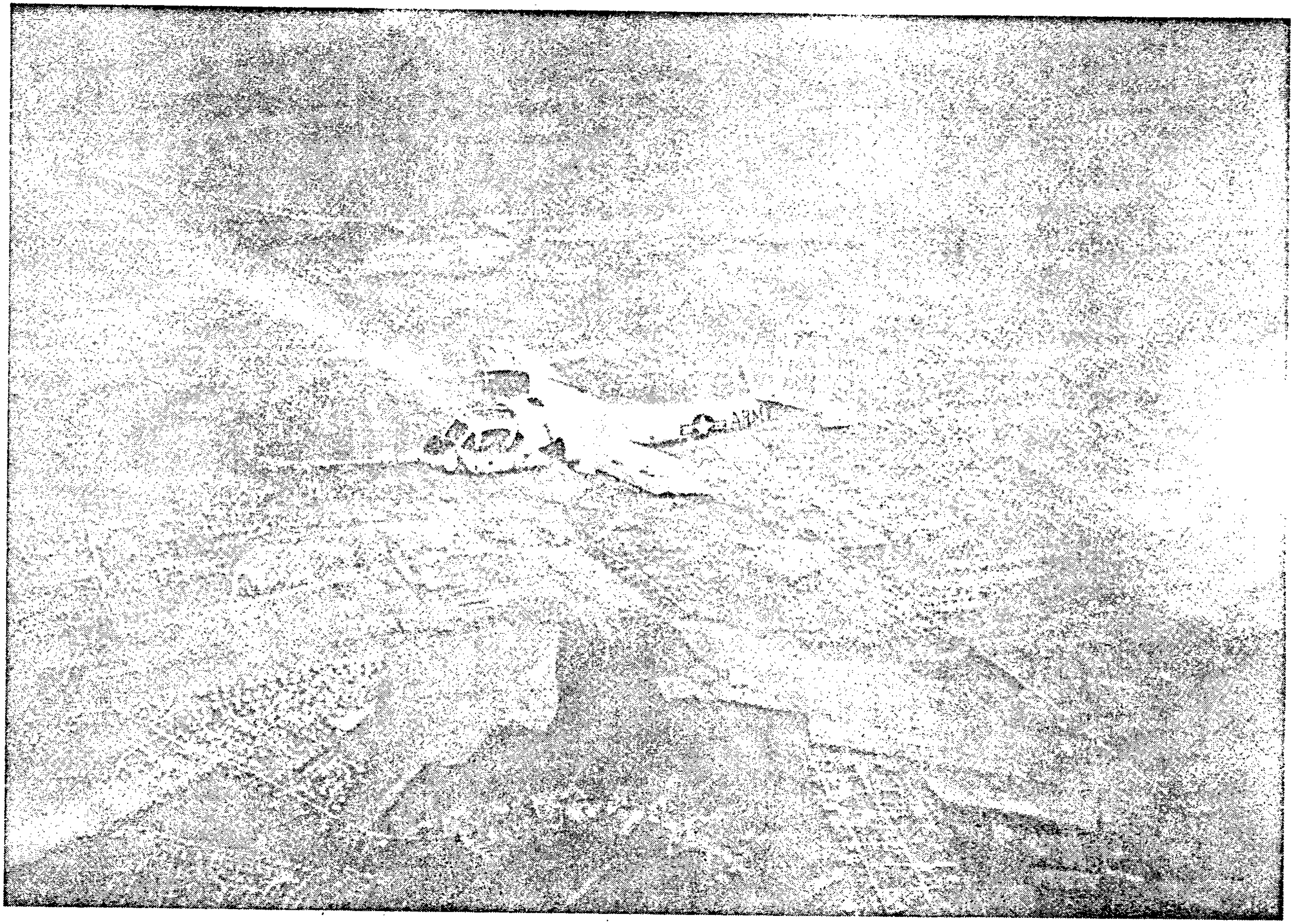
If the Communists ever use chemicals against us, we must be prepared to meet such an attack. Chemical Corps is working on this. On the other hand, Chemical Corps has performed recent tests which prove that nonlethal gases can incapacitate without killing, leaving no harmful after effects on humans or structures. Thus an objective can be captured without destroying needed buildings, bridges, or other man made structures. After receiving a dose of a gas of this type, humans will not react to orders or instructions, but wander around aimlessly. These gases are being investigated as possible alternatives to the massive exchange of thermonuclear weapons or the use of toxic agents.

In the Transportation Corps, they are concerned about advanced aircraft design, primarily of the type designated by "fly low, fly slow." The Army has to work close to the ground. Its vehicles have to be close to or on the ground, and much work is being done in this field.

I have a number of slides here showing some of the advanced designs - work that is in the development or prototype stage.

Slide 11

Mohawk



Slide 11

The first slide is the Army's Mohawk - reconnaissance and observation aircraft. This is a high-speed, short-take-off and landing aircraft for visual, photographic and electronic observations for shallow penetration into the enemy lines in the order of 25 to 40 miles.

Slide 12

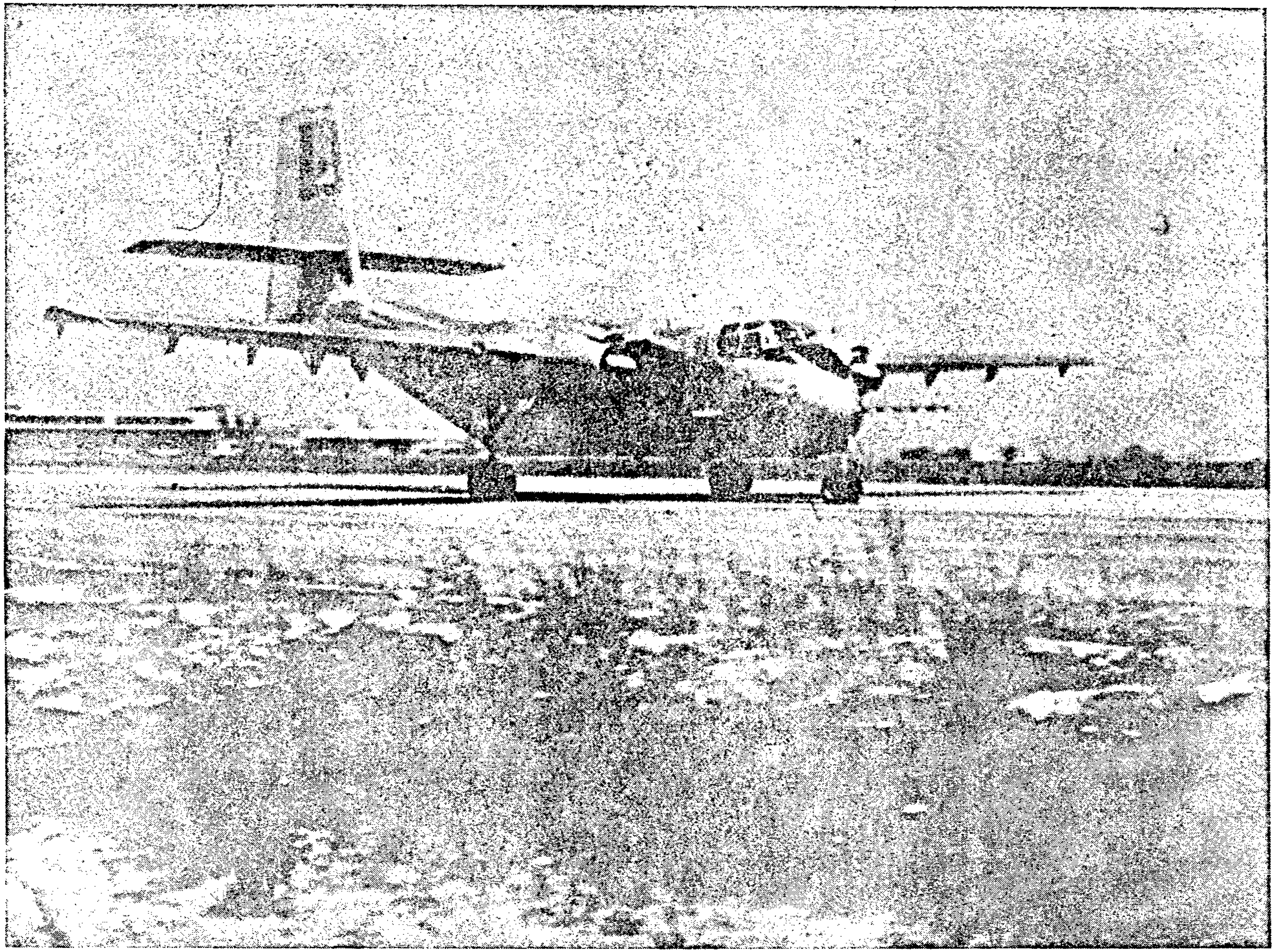
The next slide is the Caribou. This is an airplane developed by the De Haviland Company in Canada. It is a three-ton short-take-off and landing aircraft, designed primarily for civilian use in Canada, Mexico and South America but now also serving the needs of the Army. As a matter of fact, this will be a plane which could be used wherever the development of a country has not advanced to the extent where you might expect to find prepared landing fields. This is particularly true in Brazil. I learned when I was in Brazil this last summer that Focke and a large staff of Germans left Germany shortly after the close of the War, went to Brazil and are now working for the Brazilian airforce in the development of VTOL and STOL aircraft.

Slide 13

The next slide indicates work done by Goodyear on the Inflatoplane. This is a compact, rubberized craft which can be inflated by means of high-pressure CO_2. Parachuted out of aircraft, it can be assembled on the ground and flown from crude landing fields. Here it is, assembled and in the process of take-off. The engine, if you can see it, is right above the wing, a little to the right of the tail.

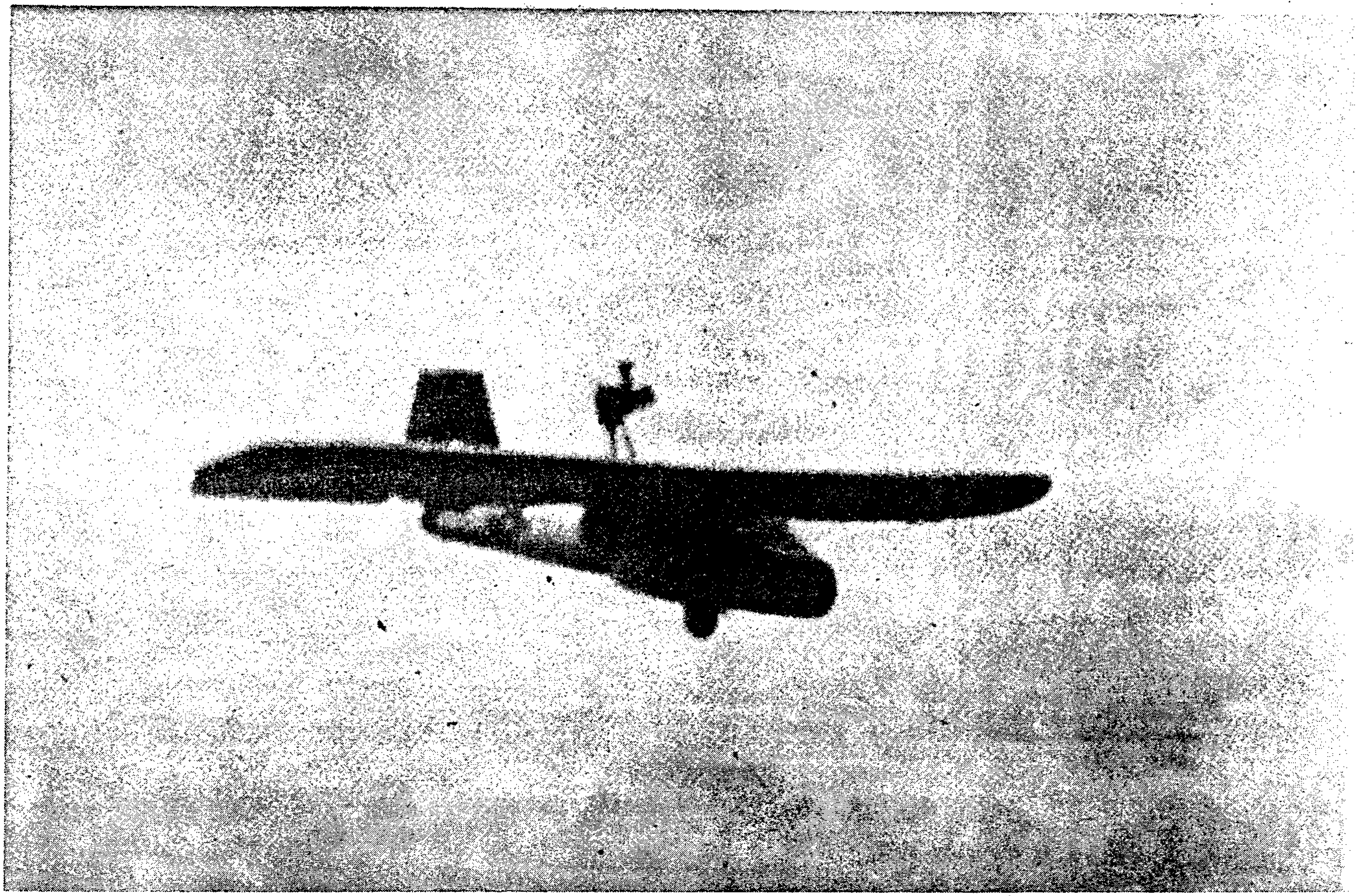
Slide 12

Caribou



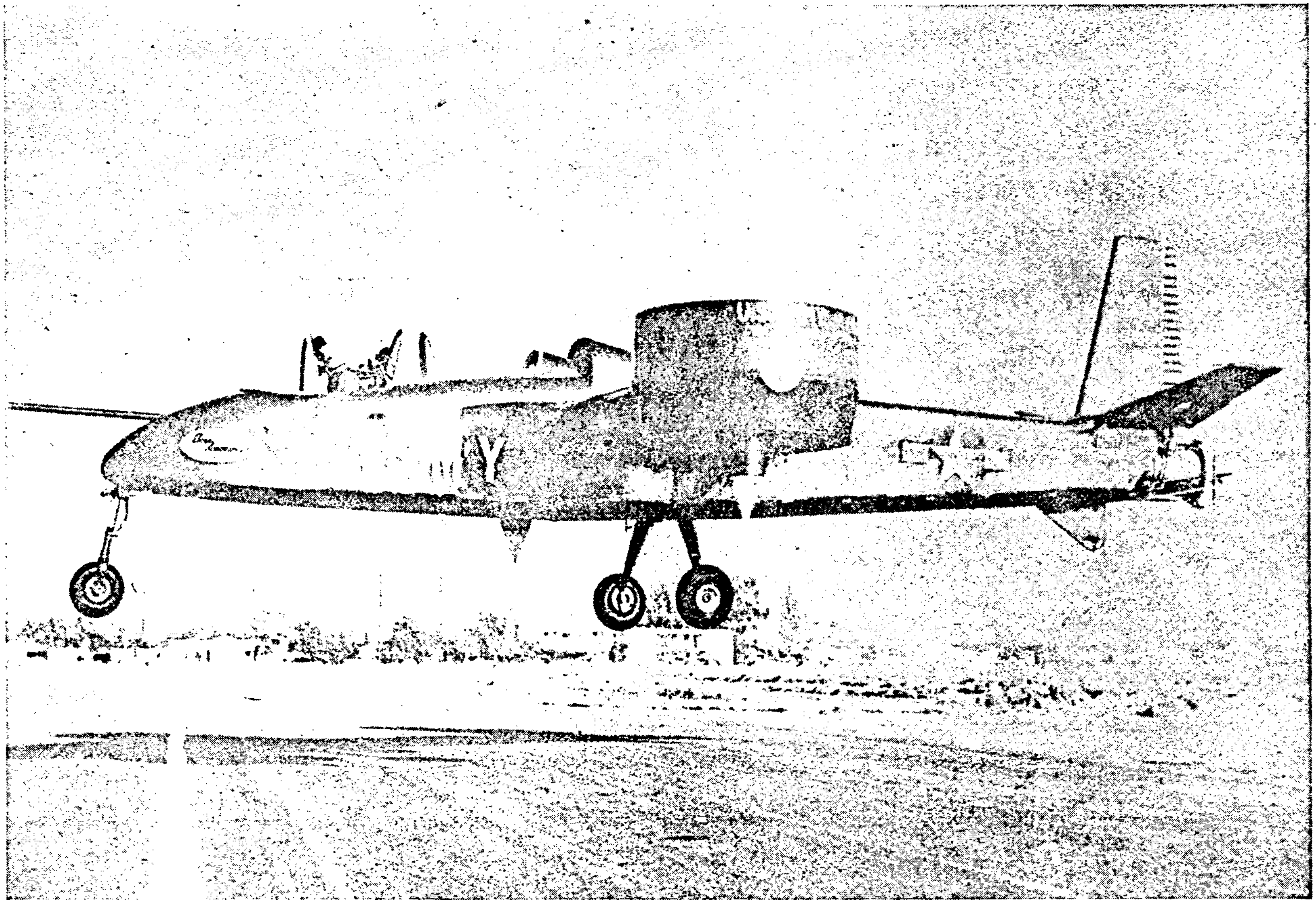
Slide 13

Inflataplane



Slide 14

Ducted Fan Doak

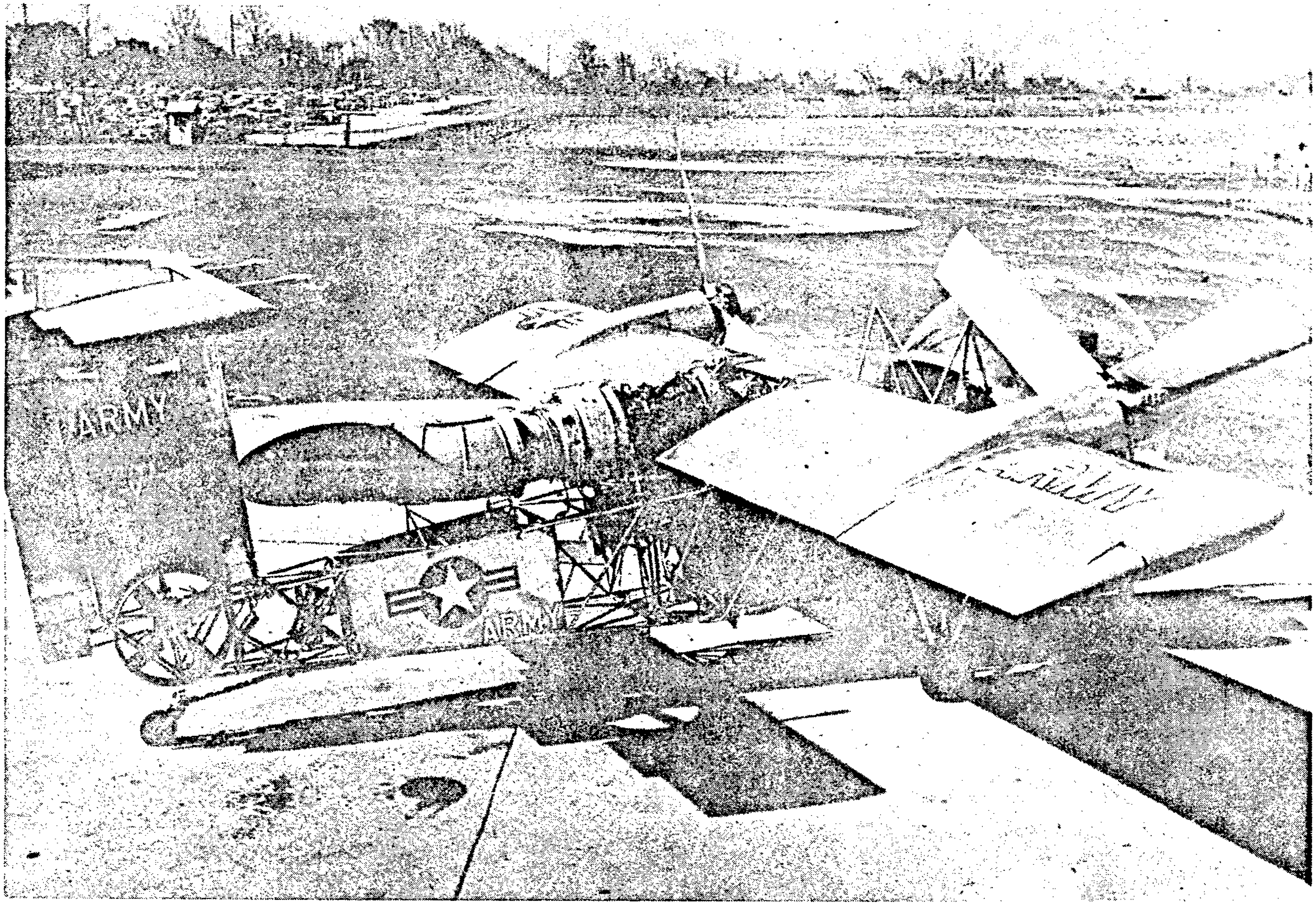


Slide 14

A little bit about the air-column supported vehicles. These are research aircraft. When I talk about air-supported, I mean the ducted fan and convertor type planes. They are of the vertical take-off and landing (VTOL) and the short take-off and landing (STOL) type aircraft. The first one is a rotatable ducted fan made by Doak. Here you can see that the fan moves through its transition phase and lift phase and then it moves forward. The problems of transition, I understand, are difficult.

Slide 15

Vertol Tilt Wing



Slide 15

The next slide is of the Vertol tilt-wing aircraft; here the whole wing tilts instead of just the fan nacelle.

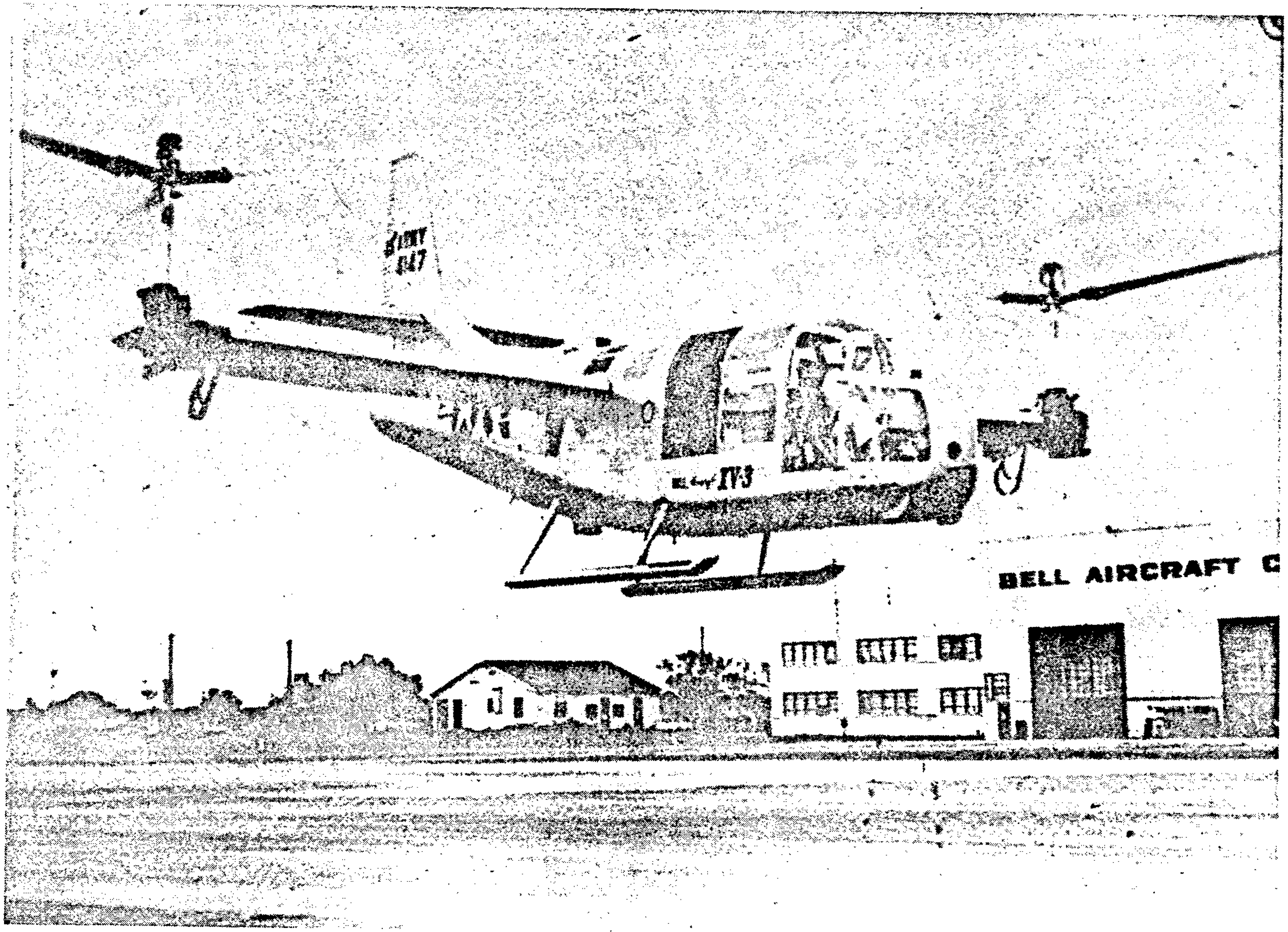
Slides 16, 17

In the converti-plane made by Bell Aircraft you can see, in the next slide, the lift phase, and in the next, the forward flight phase where the propeller has turned through 90 degrees. They have successfully gone through the transition phases of several of these experimental aircraft.

Now, something about aerial vehicles. These are general purpose vehicles. There are a number of slides I am going to show of various designs.

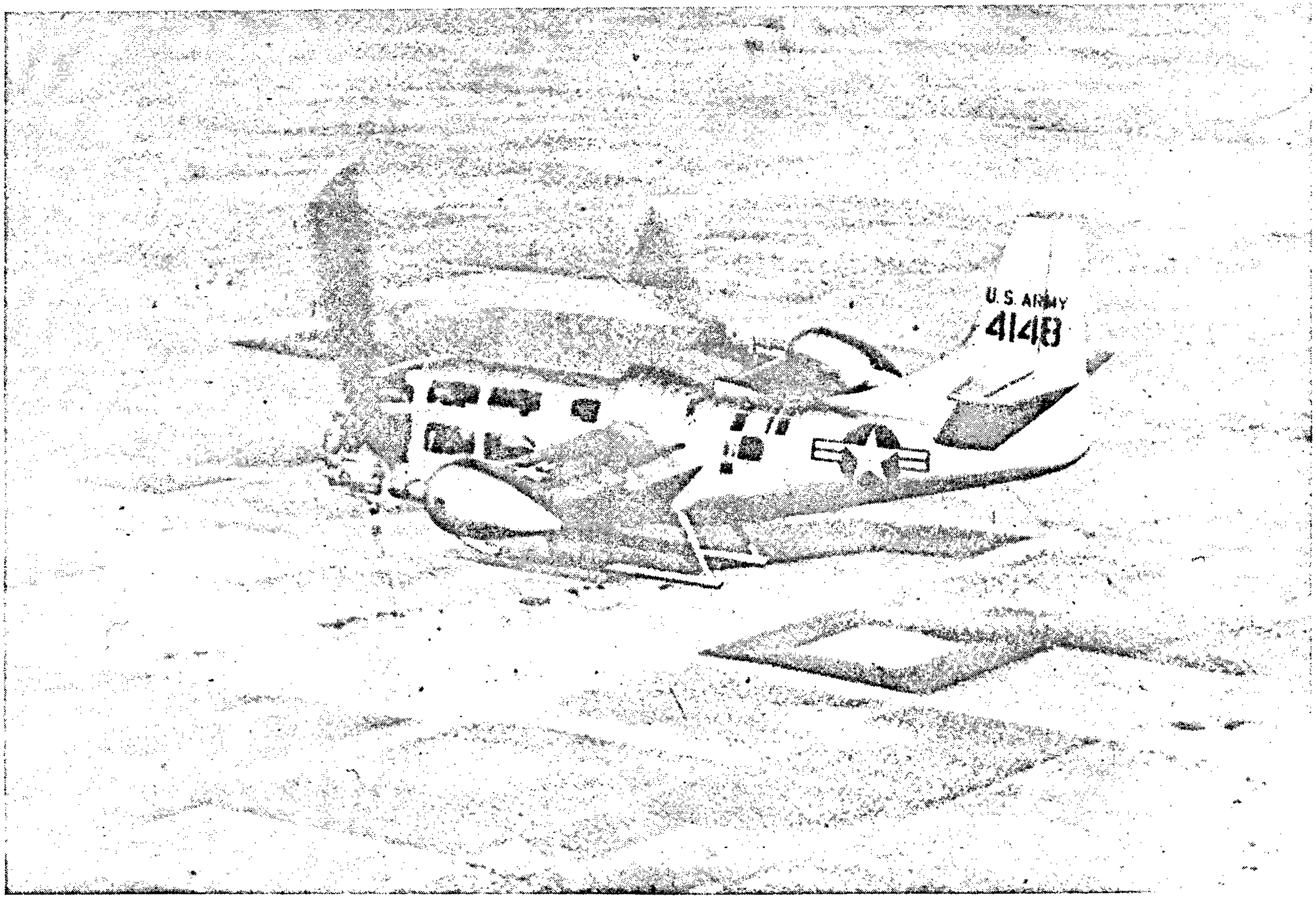
Slide 16

Bell Convertiplane - Lift Phase



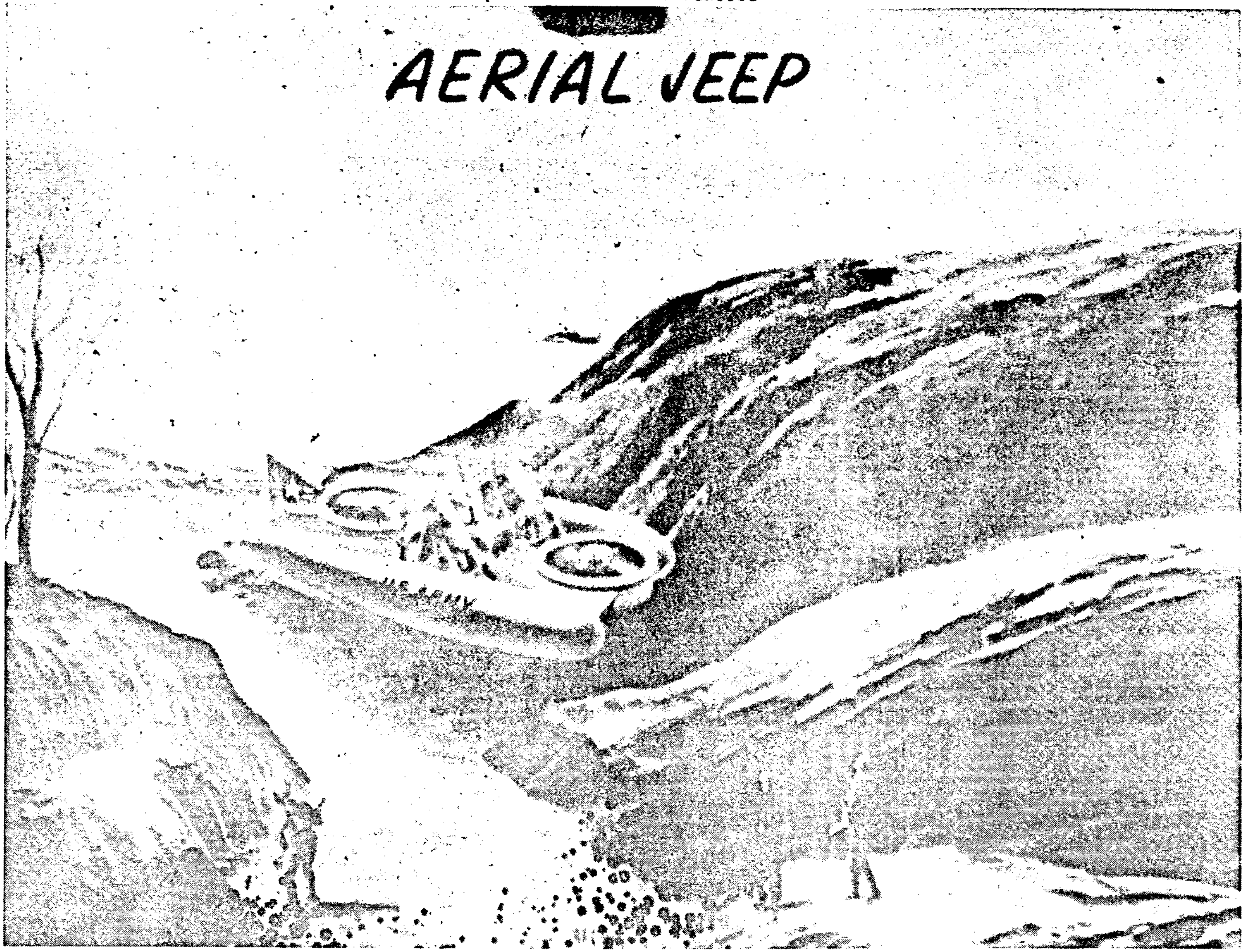
Slide 17

Bell Convertiplane - Forward Phase



Concept Phase Assault Vehicle

[redacted]



Slide 18

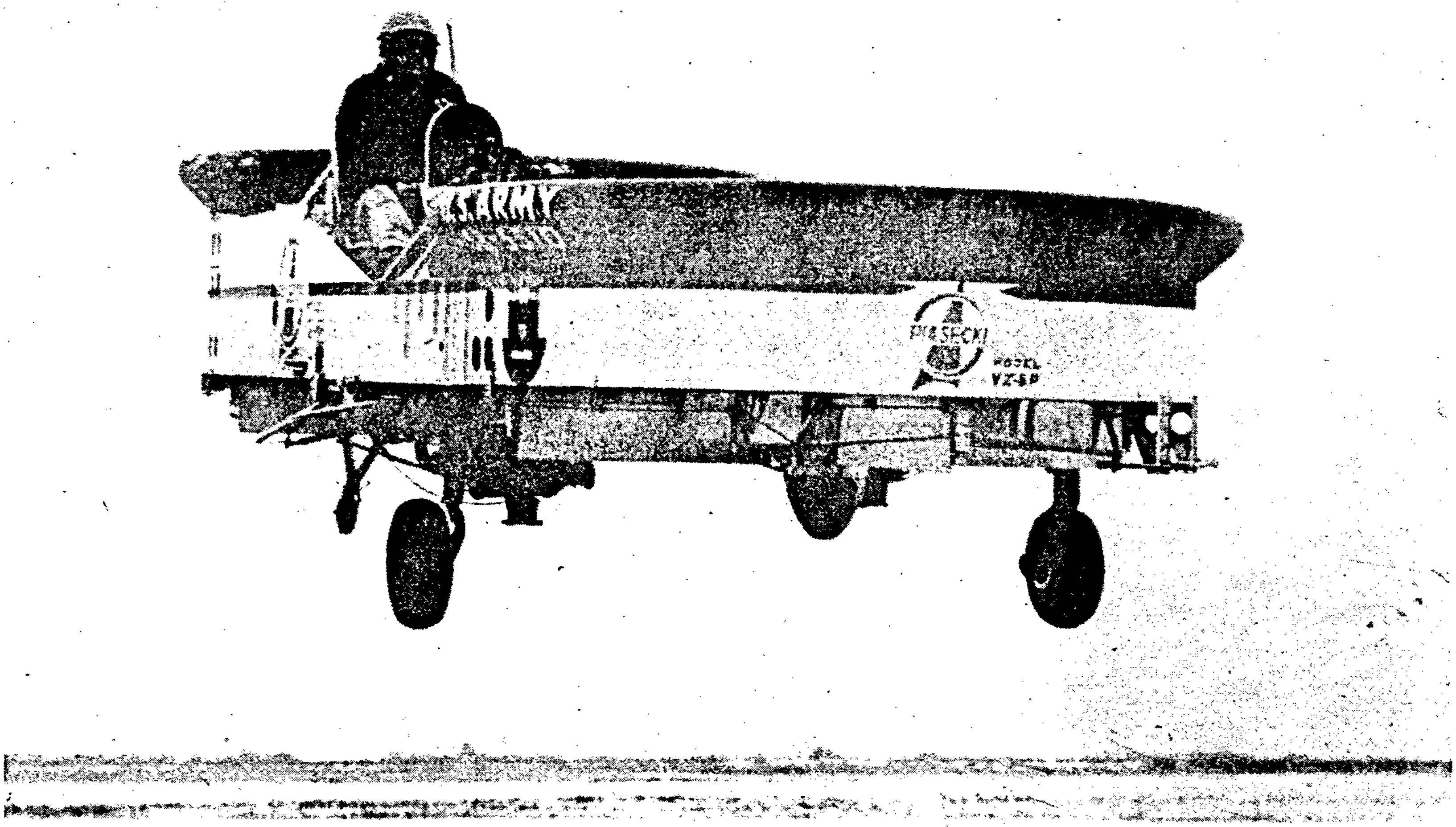
First is a Piasecki prototype aerial vehicle we used to call the "Jeep," however that is a patented name now, so we have to call it something else. It's just a flying vehicle - an artist's concept, as a matter of fact.

Slide 19, 20

The next slide is the developmental aircraft in flight and in the process of take-off, and the following slide is the same aircraft - that is Mr. Piasecki flying it and coming down to a landing. Someone has said that this vehicle has the glide angle of a rock, and I would guess that it probably has, if the engine failed.

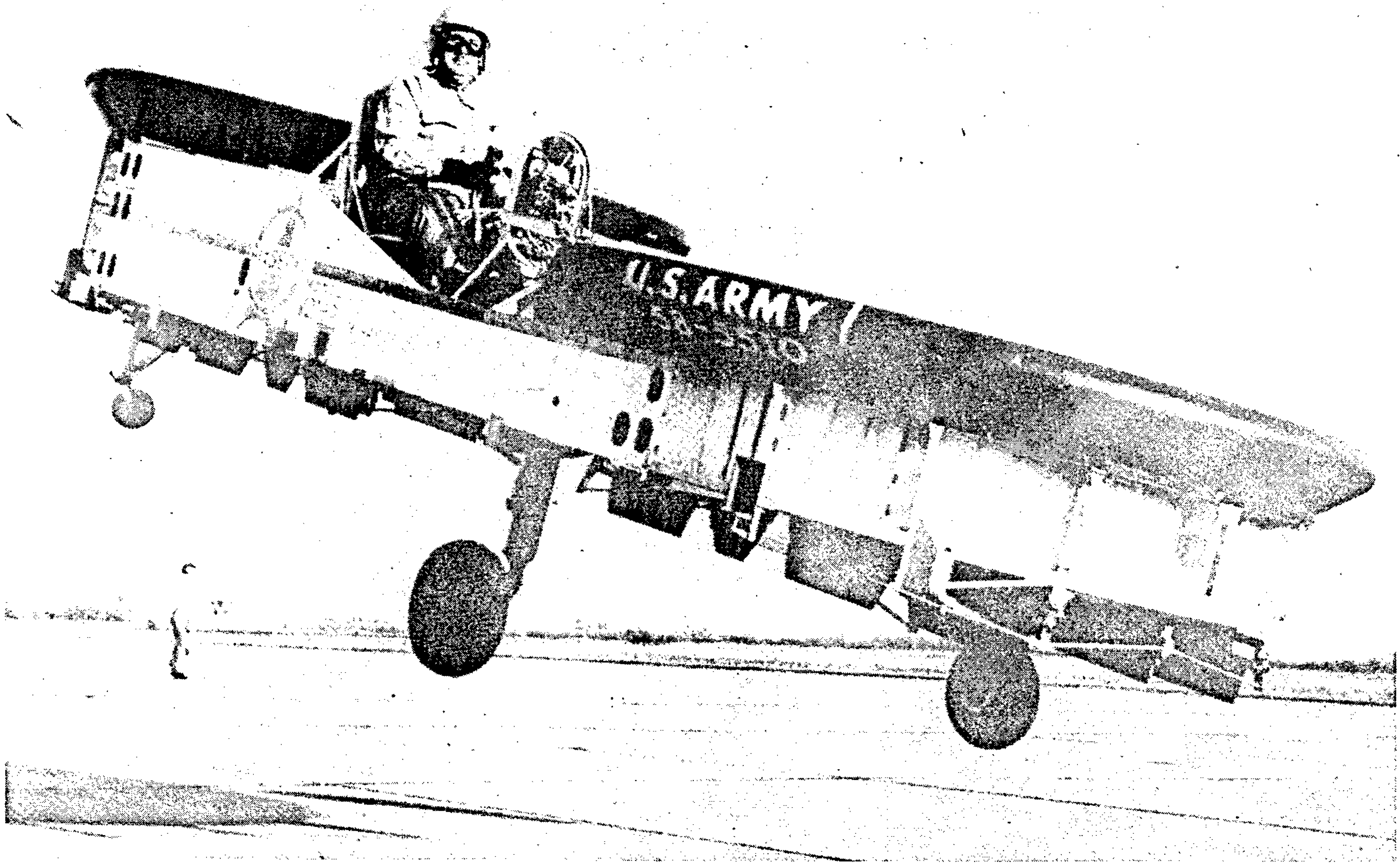
Slide 19

Aerial Jeep - Flight



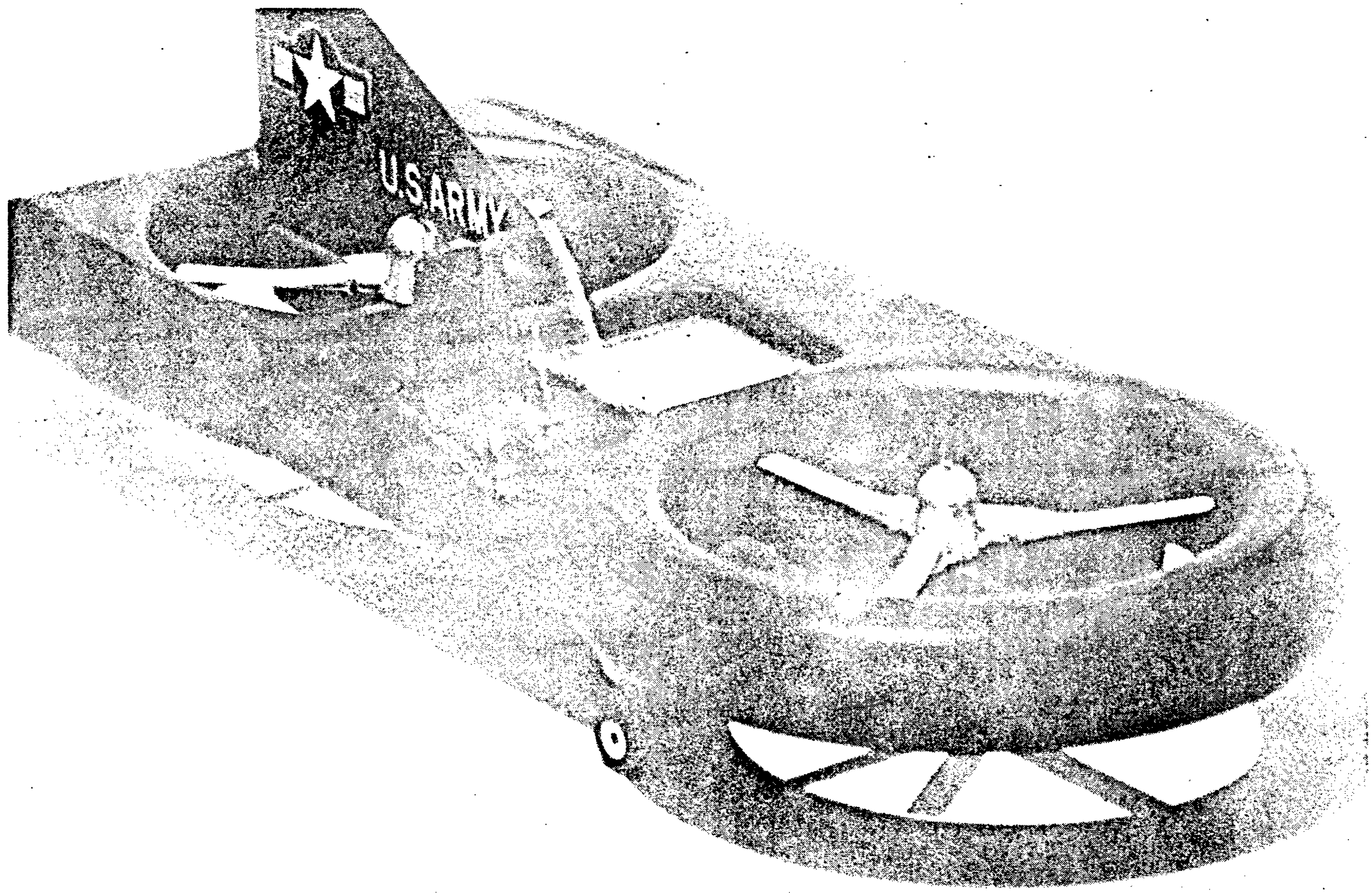
Slide 20

Piasecki - Aerial Jeep - Landing



Slide 21

Aerial "Jeep"



Slide 21

The next slide is a concept of the operational phase of an aerial jeep showing two down draft propelling systems and some idea of how it might make use of terrain cover.

I must mention here that the fly-low-fly-slow type of philosophy is governing the design and development of Army aircraft. It is designed to hug close to the earth and make use of all natural terrain so that problems having to do with observation can be carried out without danger of hazard from the enemy.

You know that the Army, and in particular the Ordnance Corps, has a major program in rockets. I thought you would like to get a look at three of them.

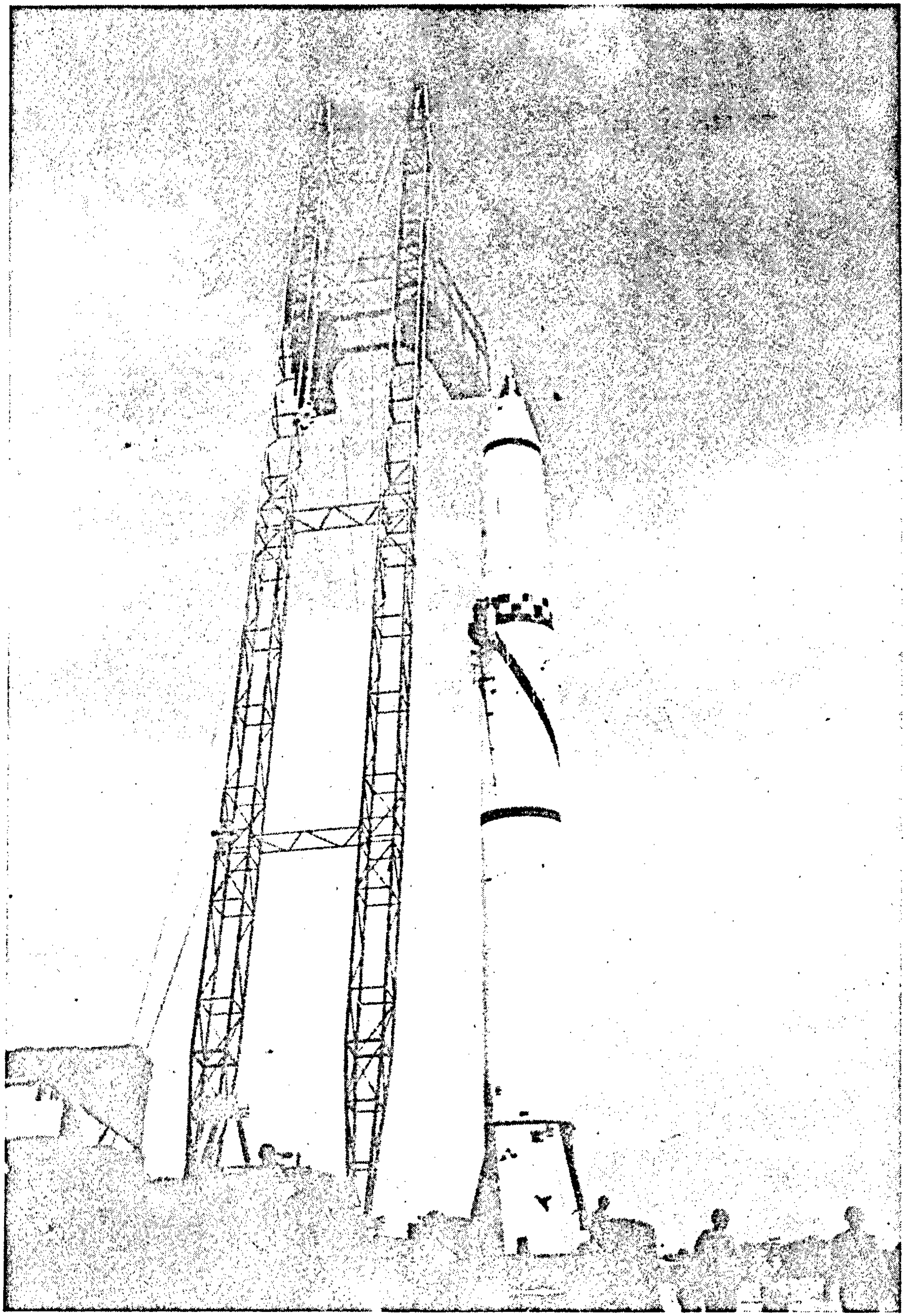
Slide 22

The first one is the Redstone, which, as you know, has proven its reliability. I do have a statement here which is the only one that the Army has been permitted to make in regard to space. "This missile (the Redstone) because of its proven reliability and stability, will be used to launch the first American into space as part of the NASA's Project Mercury." That is the extent to which I can say anything about it.

Slide 23

This is Jupiter C, or Explorer I, as you know it. It is a modified Redstone. It was the Free World's first satellite, and is expected to be up four or five years more. Judging from the number of successful firings it looks as though successful satellites are getting to be old hat.

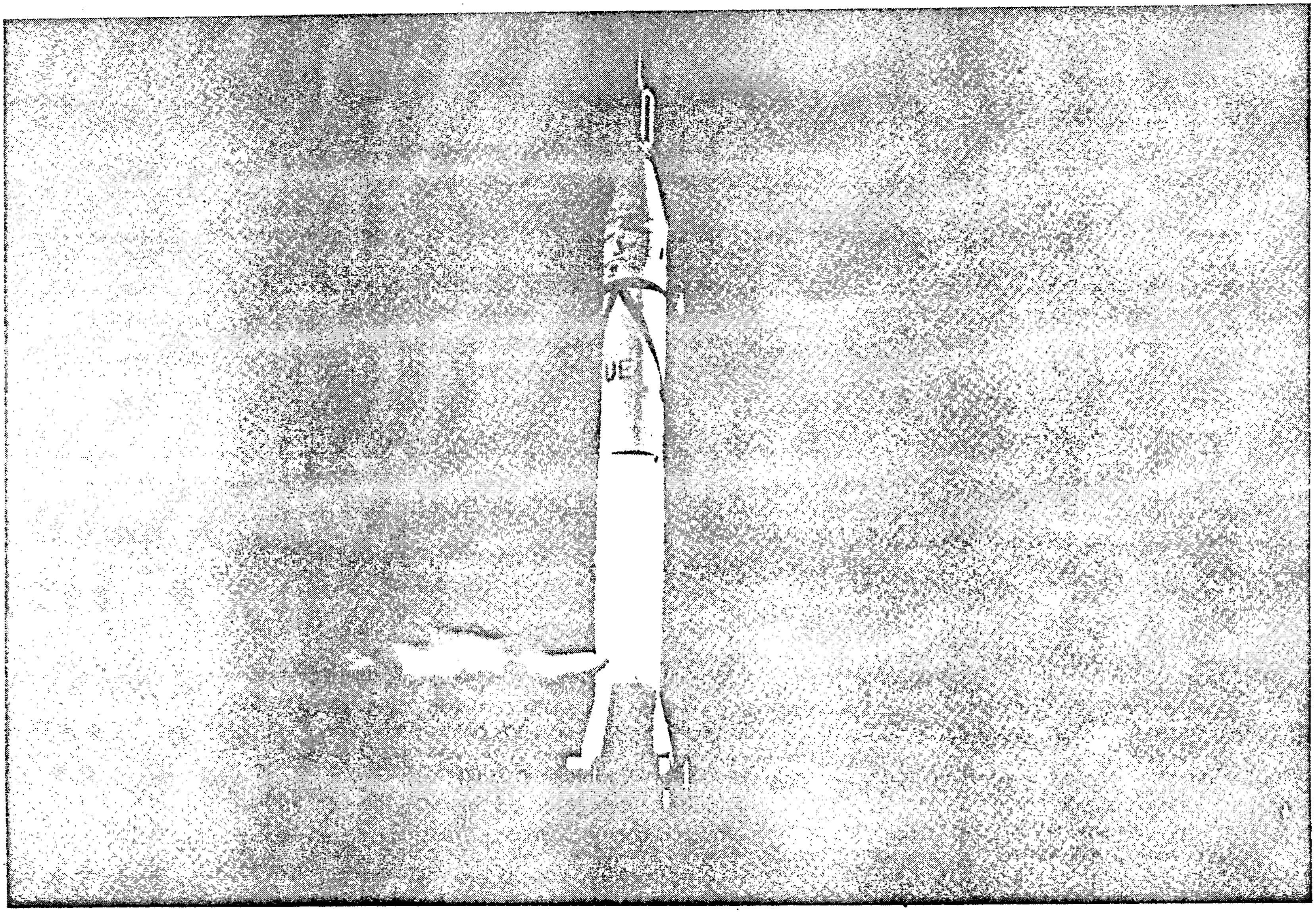
Slide 22



Redstone

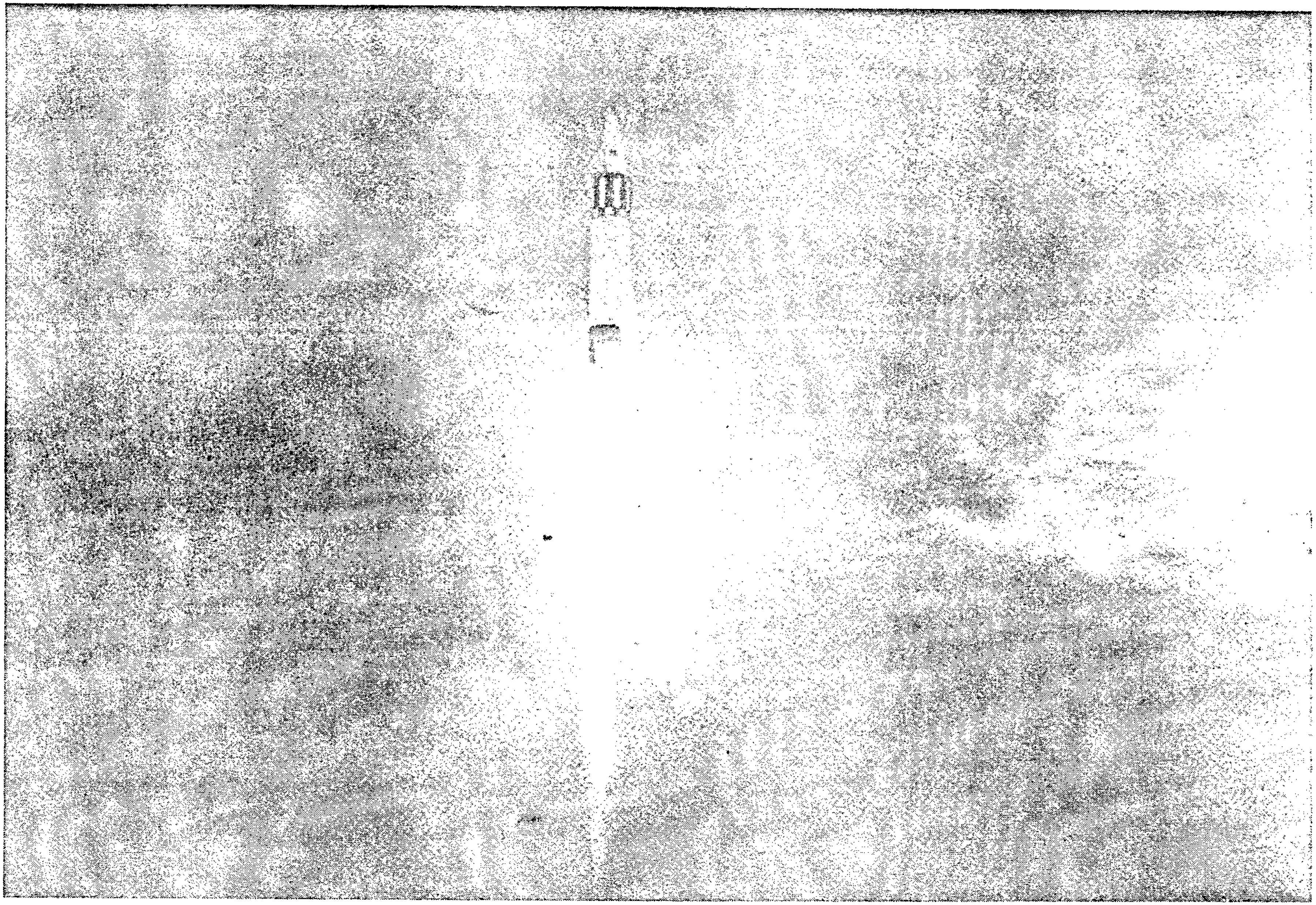
Slide 23

Jupiter C Explorer I



Slide 24

Jupiter



Slide 24

Next is the Army's IRBM Jupiter, and that covers the rocket family.

I want to say something now about contributions in the field of communication. Certainly in the field of transistors, printed circuits and miniature and micro-miniature components, we have entered into a new age in electronic packaging. Truly, this type of component will provide entirely new types of electronic instrumentation. The last I heard was that miniaturization had got down to a point where there are in the order of 600 thousand to a million parts per cubic foot component density, and I also understand that solid state materials are being used actually to build circuits - that is, inductors, resistors, and capacitors - right into the materials; so much can be expected in this area in the future.

Slide 25

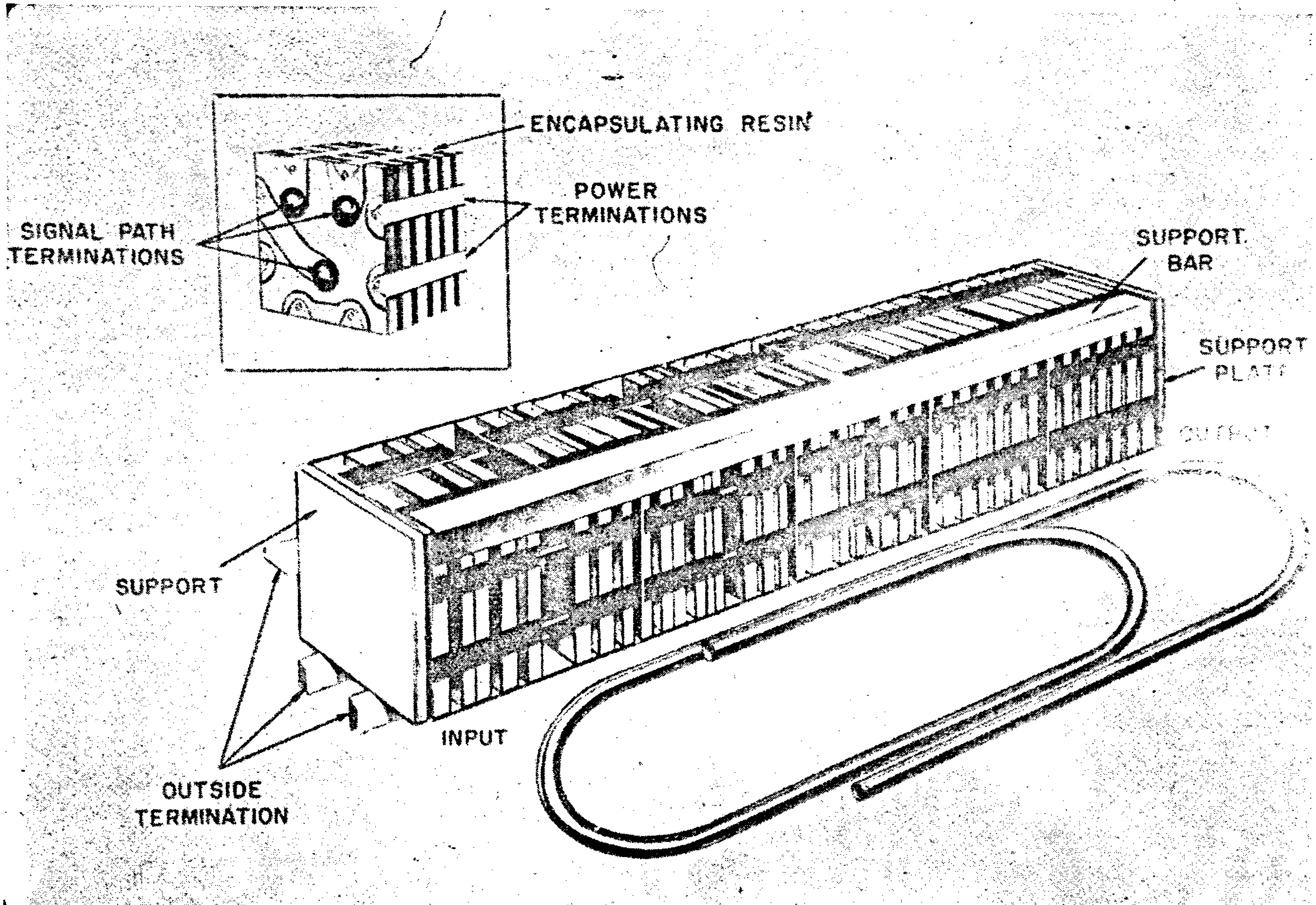
I have several slides which show some of these. The first slide is a miniaturized component as you see it along side of a paper clip. This isn't just one component - it has the entire circuit built inside of it and performs functions such as switching, oscillation and amplification.

Slide 26

The next slide is a picture of a micro-module shown alongside a lump of sugar. There is a stack of circuit elements designed for various functions in the module.

Slide 25

Micro modules



Slide 26

Micro module